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High-precision frequency measurements in the THz spectral region using an unstabilized femtosecond laser

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We perform high-precision frequency measurements in the THz frequency range using an unstabilized femtosecond laser. A simple and flexible algorithm is used to correct the beating signal resulting from the THz source and one comb line of the rectified optical comb for fluctuations of the laser repetition rate. Using this technique, we demonstrate an accuracy of our measurement device as high as $(9 \pm 3) \cdot 10^{-14}$ for the measurement of a 100 GHz source. This is *two orders of magnitude better* than previous precision measurements in this frequency range employing femtosecond lasers. © *2011 American Institute of Physics*. [doi:10.1063/1.3640234]

During the last decade, research and development in THz technology has intensified significantly. This situation holds for many different scientific disciplines, from astronomy over medicine and biology to material science.[1] Especially in communication technology, the frequency range above 100 GHz has attracted considerable attention recently.[2] To foster this development, appropriate frequency measurement systems are required which offer a high-precision measurement capability over a large frequency range and are easy to use and cost-effective.

So far, there have been several attempts to fulfill these requirements. Based on the well-established optical frequency comb,[3,4] a technique for the measurement of free-space THz radiation has been introduced.[5–9] Rectifying the optical frequency comb, a down-converted (and offset-free) frequency comb is obtained covering the whole frequency range up to several THz. Such a THz frequency comb outperforms local oscillators which can usually only be used over a limited frequency range. Common to the high-precision measurement techniques demonstrated so far is a stabilized laser system, with which a frequency accuracy of 10^{-11} has been achieved in the THz frequency range.[5–7]

In this paper, we demonstrate high-precision frequency measurements of free-space THz radiation using an unstabilized frequency comb. By simultaneously measuring the fluctuations of the laser repetition rate, we correct the THz measurements for these fluctuations and achieve a frequency accuracy of $9 \cdot 10^{-14}$. This is two orders of magnitude better than previously reported results using femtosecond lasers.[5,7] Along with this considerably improved accuracy our measurement and analysis technique simplifies the experimental setup. This is because it enables THz frequency measurements using an arbitrary optical frequency comb, independent of its stabilization ability. Moreover, our software-based correction algorithm offers a high flexibility concerning the input signals and acts as a versatile tool for data analysis for which one usually needs different devices like frequency counters, spectrum analyzers, mixers, and phase-lock loops.

A schematic diagram of the measurement setup is shown in Fig. 1(a). A pulsed Ti:Sa laser (repetition rate $f_{\text{rep}} \sim 76$ MHz, pulse length 100 fs, center wavelength 810 nm, spectral width 20 nm) without frequency stabilization is used to generate an optical frequency comb. The optical beam with an average power of 10 mW is focused onto a photoconductive antenna (PCA), bowtie-shaped contact pads on low-temperature-grown GaAs, excitation gap 6×6 μm^2). The optical comb is rectified in the PCA due to the generation of free carriers. This offset-free frequency comb covers the frequency range from DC to approximately 3.5 THz. The latter value is obtained from measurements of the free-carrier life time using a standard pump-probe setup. This THz comb acts as a detector and mixer for an incoming THz electromagnetic wave. A current flow between the contact pads of the PCA is induced which results from the beating signals of the electromagnetic wave with the adjacent comb lines. After amplification (Femto Messtechnik GmbH, HCA-40M-100K-C, 40 MHz bandwidth, 10^5 V/A transimpedance gain), the current is detected with an analog-to-digital converter (ADC, National Instruments, NI-5122, 14 bit resolution, 10 MHz sampling rate), see Fig. 1(a). The lowest-frequency component of the measured beating signal is given by the frequency difference between the incoming cw-THz wave (f_{THz}) and the nearest (m-th) comb line, $f_{\text{b,THz}} = |m \cdot f_{\text{rep}} - f_{\text{THz}}|$. In our experiments, the free-space cw THz radiation is generated by

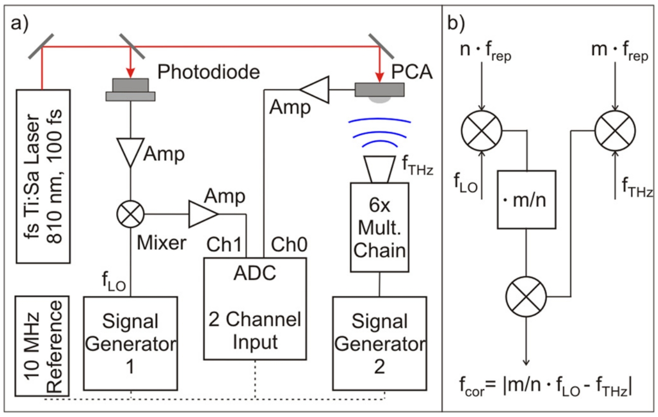

FIG. 1. (Color online) (a) Schematic diagram of the THz measurement setup (Amp = Amplifier, PCA = photoconductive antenna, ADC = analog to digital converter, Ch0/Ch1 = input channels of ADC) and (b) principle of the transfer concept.

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an active frequency-multiplier chain (Radiometer Physics FE-110, multiplication factor 6, output power ~10 dBm), set to an output frequency of $f_{\mathrm{THz}} = 100.02$ GHz.

In an unstabilized laser system the cavity length of the laser changes due to thermal effects, which in turn causes fluctuations of the laser repetition rate. This has a strong influence on the measurement of $f_{\mathrm{b,THz}}$. Usually this problem is eliminated by stabilizing the frequency comb. However, it is also possible to use an unstabilized frequency comb for frequency measurements and correct these measurements for the repetition rate fluctuations, which have been independently measured. By this method the frequency comb is only used to transfer the frequency accuracy of a local oscillator to another frequency range, thus being referred to as transfer concept.[10] We use this transfer concept, which has originally been demonstrated for optical frequency measurements,[10] for the construction of a software-based correction algorithm and apply it to precision measurements of THz frequencies. For the measurement of the repetition rate of the laser, we first rectify the optical comb using a fast photodiode. The n-th comb line of the photodiode signal is then mixed with a local oscillator, set to a frequency f_{LO}, yielding a beat signal with the lowest frequency contribution at $f_{\mathrm{b,rep}} = |n \cdot f_{\mathrm{rep}} - f_{\mathrm{LO}}|$. To detect the signal at $f_{\mathrm{b,rep}}$ we use the same two-channel ADC that is employed for the measurement of $f_{\mathrm{b,THz}}$, see Fig. 1(a). To correct the THz beating signal for fluctuations of f_{rep} one has to multiply the frequency of $f_{\mathrm{b,rep}}$ with a factor m/n and mix this signal with the THz beating signal $f_{\mathrm{b,THz}}$. The difference frequency of this mixing process is then given by

$$f_{\mathrm{cor}} = |m/n \cdot f_{\mathrm{b,rep}} \otimes f_{\mathrm{b,THz}}| = |m/n \cdot f_{\mathrm{LO}} - f_{\mathrm{THz}}|, \quad (1)$$

where the repetition rate of the laser has been cancelled out.

To provide a common time base for the measurement of $f_{\mathrm{b,THz}}$ and $f_{\mathrm{b,rep}}$, a 10 MHz quartz oscillator is used. By this, source and detector are synchronized to the same reference and fluctuations of the reference frequency will cancel out. We emphasize that such a closed system, as already used in Refs. 5 and 7, yields information about the frequency accuracy of the setup that can be used for the measurement of absolute frequencies. We show further below that this accuracy for our experimental setup is $9 \cdot 10^{-14}$. For absolute frequency measurements, in which the frequency of the free-space THz source is unknown we can reference the setup to any frequency standard available, for example to a Global Positioning System-based standard or one of Physikalisch-Technische Bundesanstalt's atomic clocks with a frequency uncertainty of $2 \cdot 10^{-15}$ over 10 s.[11]

In Fig. 2(a), the normalized spectral power of the beating $f_{\mathrm{b,rep}}$ between the $n = 46$th harmonic of f_{rep} and a local-oscillator signal set to a frequency of $f_{\mathrm{LO}} = 3.4961$ GHz is plotted. The spectrum is obtained from a time-domain measurement over a period of 10 s. A Gaussian approximation of the beat signal shows a center frequency of 114964 Hz and a full width at half maximum (FWHM) of 21 Hz. The change of the repetition rate over the measurement time of 10 s can be visualized using the well-known concept of instantaneous frequency.[12] First we generate a complex representation $z(t)$ of the measured data using the Hilbert transform. This yields a complex time-domain signal composed of the original signal (as the real part) and a signal shifted by 90° (as the imaginary part). These two components are shown in Fig. 2(b) for the first 50 μs of our 10 s long time trace. A derivation of the phase of $z(t)$ after time[13] yields the instantaneous frequency of the beating signal $f_{\mathrm{i,beat}} = 1/(2\pi)\, \mathrm{d}arg[z(t)]/\mathrm{d}t$, from which the instantaneous frequency of the repetition rate is easily derived: $f_{i,rep} = |f_{\mathrm{i,beat}} - f_{\mathrm{LO}}|/n$. In Fig. 2(c), the deviation of $f_{\mathrm{i,rep}}$ from its mean value is shown for the first 0.1 s with the mean value having been obtained from the complete 10 s long time trace. The observed fast fluctuations as well as the long-term drift are the result of various technical and fundamental noise sources of mode locked Ti:Sa lasers.[14]

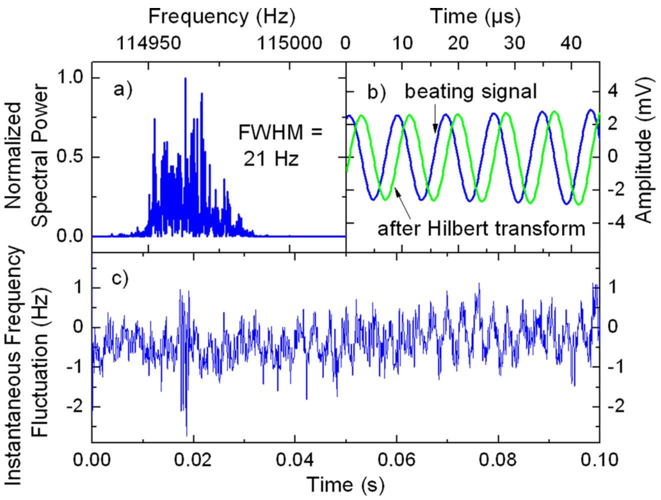

FIG. 2. (Color online) (a) Fourier transform of the 46th harmonic of the repetition rate measured over 10 s. (b) First 50 μs of the corresponding time-domain trace (beating signal) and its Hilbert transformed signal. (c) Deviation of the instantaneous frequency of the repetition rate within the first 100 ms from its average value measured over 10 s.

For the measurement of free-space THz signals, the repetition rate fluctuations lead to a significant modification of the beating peaks. This is visualized in Figs. 3(a) and 3(b), where two signals are shown which arise from a beating between our THz signal at 100.02 GHz and the $m = 1316$th comb line of the THz frequency comb. Again, the plots show

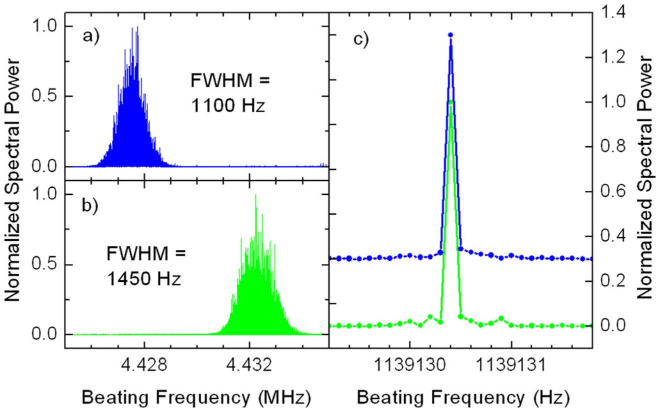

FIG. 3. (Color online) (a),(b) Fourier transforms of two measured THz beating signals. The repetition rate of the fs laser changed by ~ 4 Hz between the two measurements leading to a shift of ~ $m \cdot 4$ Hz between the beating signals. (c) Corrected beating signals of (a) and (b) with a resolution-limited FWHM (0.1 Hz frequency spacing). For clarity, one signal has been shifted vertically.

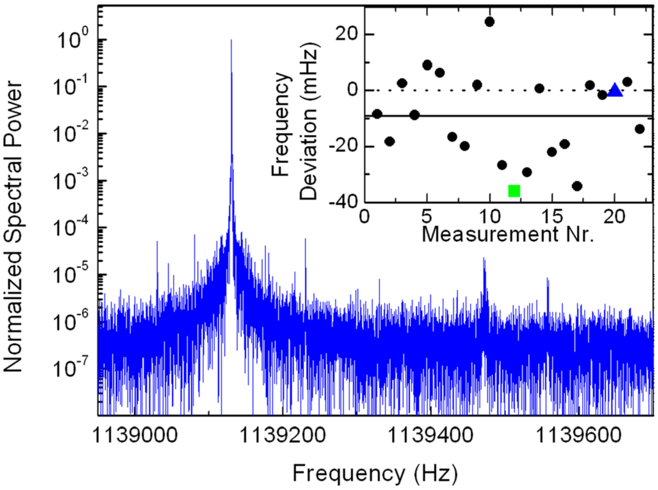

FIG. 4. (Color online) Semi-logarithmic plot of the corrected beating signal shown as upper curve in Fig. 3(c). The inset shows the frequency deviation between the adjusted and the measured frequency of 100.02 GHz for 22 measurements. The resulting mean value of −0.009 Hz is represented by a black line. The triangle and the square in the inset correspond to the upper, respectively lower, data set shown in Fig. 3.

the normalized power spectra of two 10 s long time trace, having been measured with a time delay of approximately 1 min. Due to the higher comb mode, the influence of the repetition rate fluctuations is much more dominant than in the mixing process with f_{LO} and results in a broadening of the measured beating signal. Additionally, there is a large drift of several kHz in the center frequencies of the two beating signals. The latter corresponds to a drift of the averaged repetition rate from approximately 75,999,675.1 Hz (Fig. 3(a), first measurement) to 75,999,671.5 Hz (Fig. 3(b), second measurement). It should be noted that due to these repetition rate fluctuations the value for m can be extracted from two measurements recorded several minutes apart, by simply comparing the shift of the center frequencies of $f_{\mathrm{b,rep}}$ and $f_{\mathrm{b,THz}}$. No intentional detuning of the laser repetition rate as done in Refs. 5–7 is necessary.

To correct the THz beating signal for fluctuations of the repetition rate we have designed a simple software-based algorithm consisting of only three main steps: First, a Hilbert transformation of the measured time trace $f_{\mathrm{b,rep}}$ is performed as described before. Second, the frequency of the resulting complex time trace is multiplied by m/n. To this end we use Euler's Formula and multiply the phase values by m/n. Third, the real part of the resulting time trace is multiplied by the time trace of the THz beating signal $f_{\mathrm{b,THz}}$ to obtain the corrected signal at the difference frequency given by Eq. (1). In addition to these three steps, the measured signals have been frequency-filtered with a Gaussian filter (width >20 kHz) to suppress higher order mixing products as well as other noise signals inherent to the experimental setup. We like to emphasize that no additional fitting, interpolation or other data processing methods have been used. The main advantage of this software solution as compared to a hardware implementation of the transfer concept is the flexibility since the technique can be directly applied to different beating frequencies and no additional devices (e.g., spectrum analyzers, frequency counters, mixers, phase-lock loops) have to be used. This considerably simplifies the experimental setup.

The correction process has been applied to the data shown in Figs. 3(a) and 3(b). The result of this procedure is plotted in Fig. 3(c), showing the normalized power spectrum of the corrected THz beating signals. In both cases, the broad spectra of Figs. 3(a) and 3(b) have been reduced to a resolution-limited peak with a FWHM below 0.1 Hz, which is the frequency spacing resulting from our measurement time of 10 s. Fig. 4 shows a semi-logarithmic-scaled plot of the upper curve of Fig. 3(c) visualizing a signal-to-noise ratio of about 60 dB for the measurement time and sampling rate as discussed above. Analyzing the time traces of both signals plotted in Fig. 3(c), a software-based frequency counter yields beating frequencies of 1139130.434 Hz and 1139130.399 Hz, respectively. From these values and Eq. (1) we obtain $f_{\mathrm{THz}} = 100{,}019{,}999{,}999.999$ Hz and $f_{\mathrm{THz}} = 100{,}019{,}999{,}999.964$ Hz, having a deviation of −0.001 Hz and −0.036 Hz, respectively, from the adjusted value of 100.02 GHz. From a statistical analysis of 22 measurements (see inset of Fig. 4) we obtain a mean frequency deviation of (-0.009 ± 0.003) Hz. This yields a relative accuracy of our measurement device of $(9 \pm 3) \cdot 10^{-14}$, which is limited by the noise properties of the local oscillator (f_{LO}) and the software correction algorithm.

In summary, we have constructed a device for high-precision cw-THz measurements using an unstabilized femtosecond laser as a transfer oscillator. To correct the measured THz signal for the repetition rate fluctuations, a very simple and flexible algorithm based on digital signal processing is used. This method, which can also be used to measure multiple peaks or complex spectral shapes, allows for a measurement accuracy as high as $9 \cdot 10^{-14}$.

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