

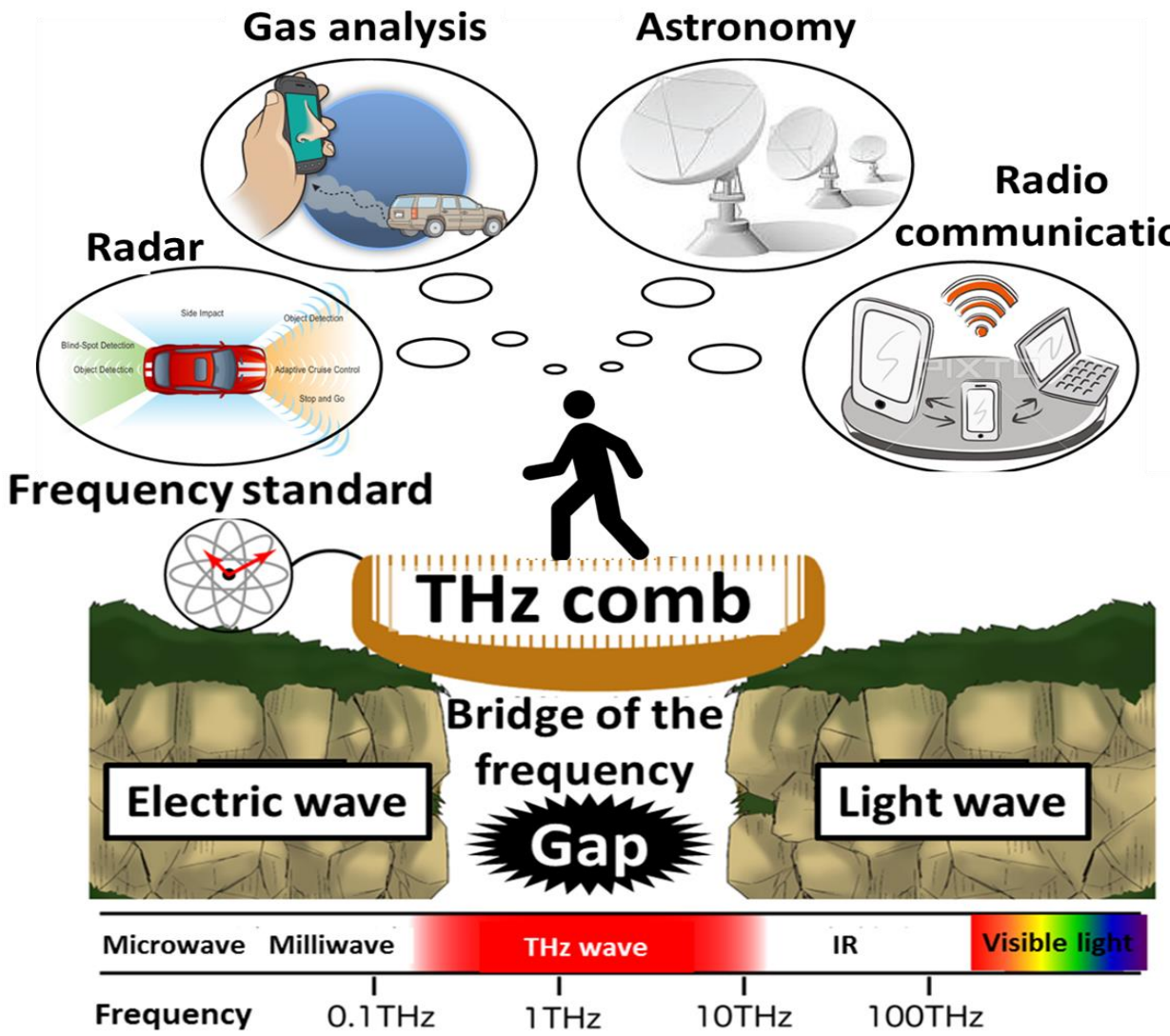
STu1F.4

Real-Time Absolute Frequency Measurement of CW-THz Wave Based on Dual THz Combs

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THz comb bridges THz gap as frequency scale



Frequency measurement of CW-THz wave

*Practical
CW-THz sources*



THz-QCL



UTC-PD

Conventional methods of the frequency measurement

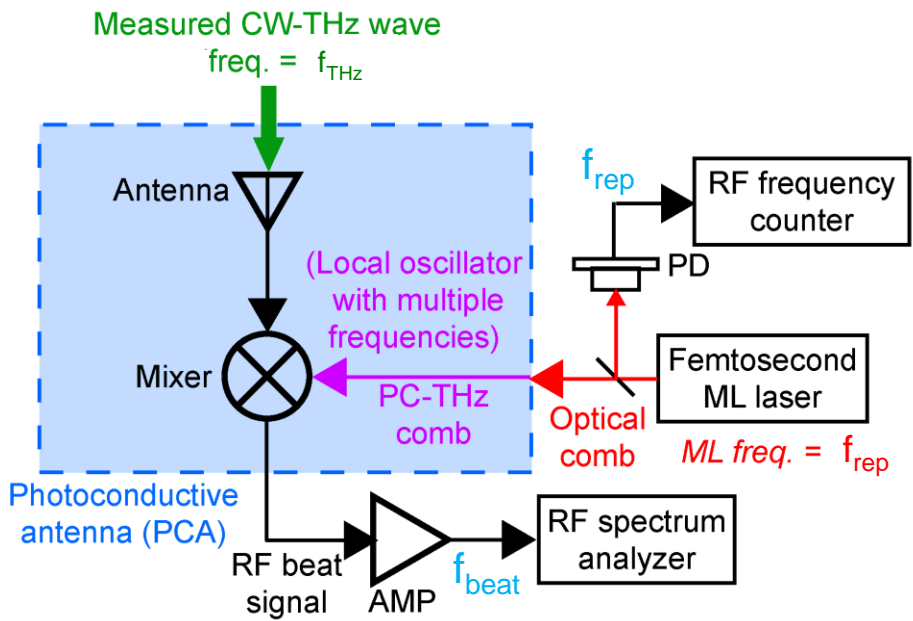
- Electrical heterodyne method
- Optical interferometric method



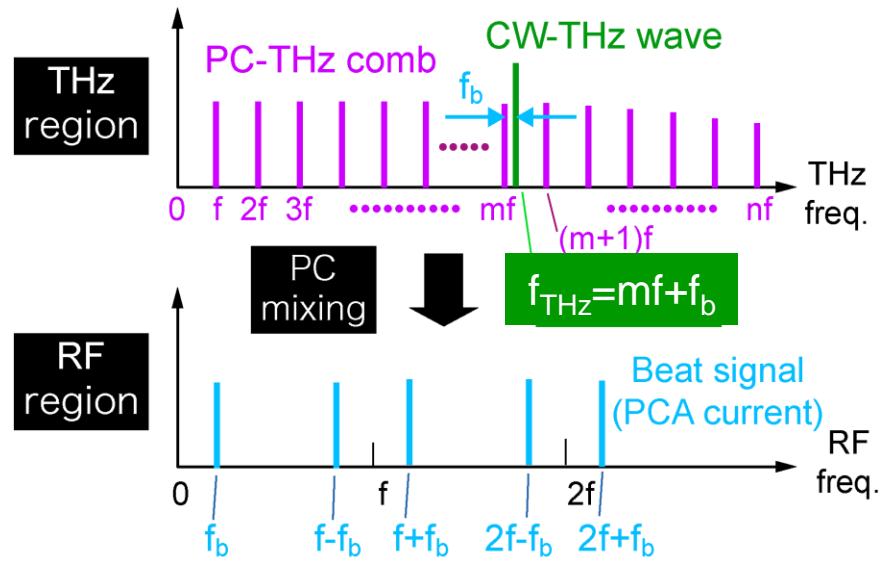
**Cryogenic cooling of mixers
or detectors is required**

**Frequency measurement of CW-THz wave without the
need for cryogenic cooling is strongly required !**

THz-comb-referenced frequency measurement



Freq. domain

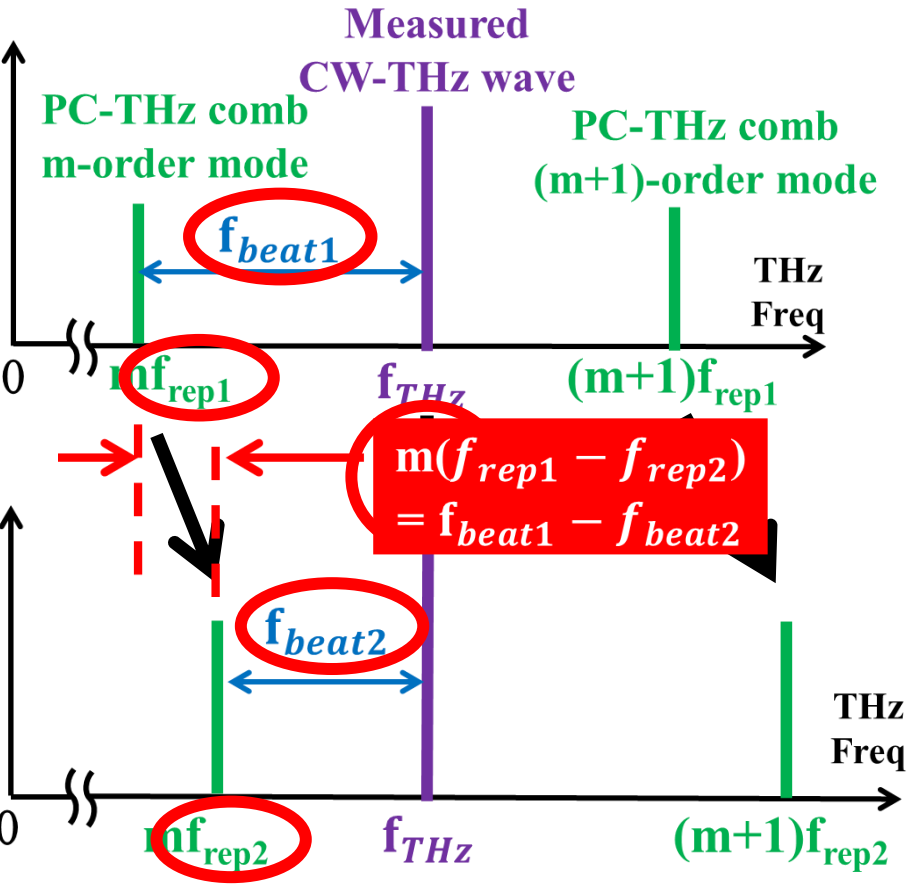


$$f_{\text{THz}} = m f_{\text{rep}} \pm f_{\text{beat}}$$

m : order of comb mode
 f_{rep} : ML frequency
 f_{beat} : beat frequency

Ref) S. Yokoyama et al, *Opt. Express* **16**, 13052-13061 (2008).
 T. Yasui et al. *Opt. Express* **17**, 17034-17043 (2009).

Determination of m and sign of f_{beat}



Measurement of f_{rep1} and f_{beat1}

$$m = \frac{|f_{beat1} - f_{beat2}|}{|f_{rep1} - f_{rep2}|}$$

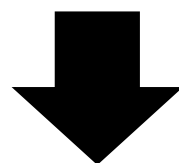
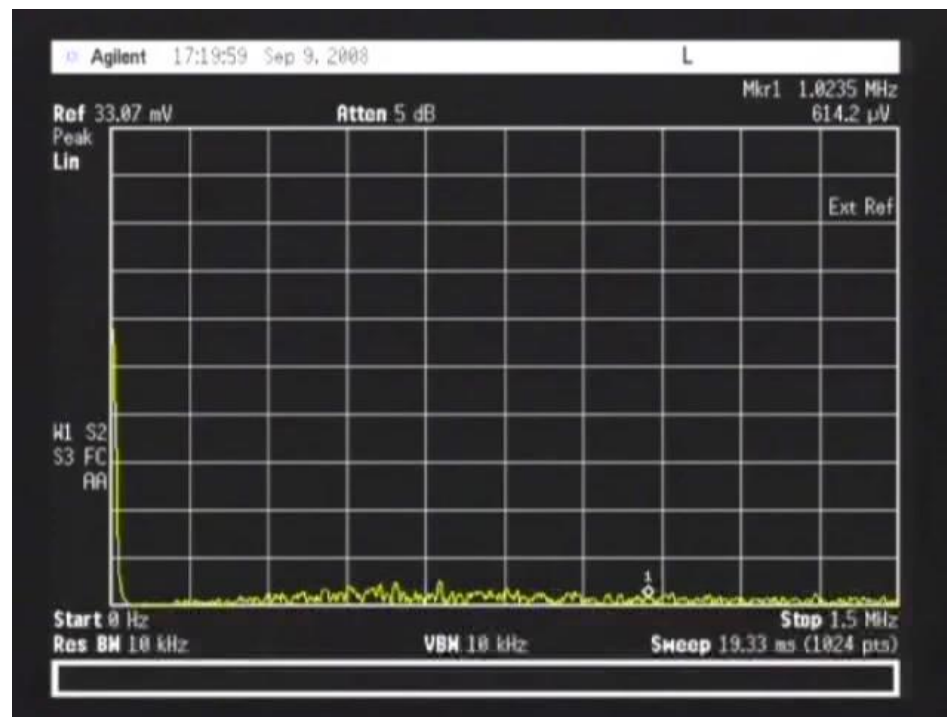
Measurement of f_{rep2} and f_{beat2}

Assumption : f_{THz} is sufficiently stable during measurements of f_{beat1} and f_{beat2}

For example

CW-THz source (UTC-PD)

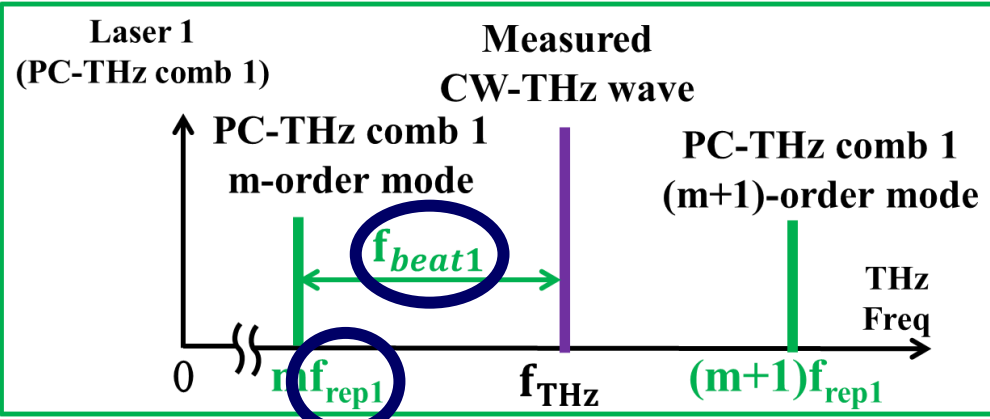
- fast frequency fluctuation
- large frequency fluctuation (mode hopping)



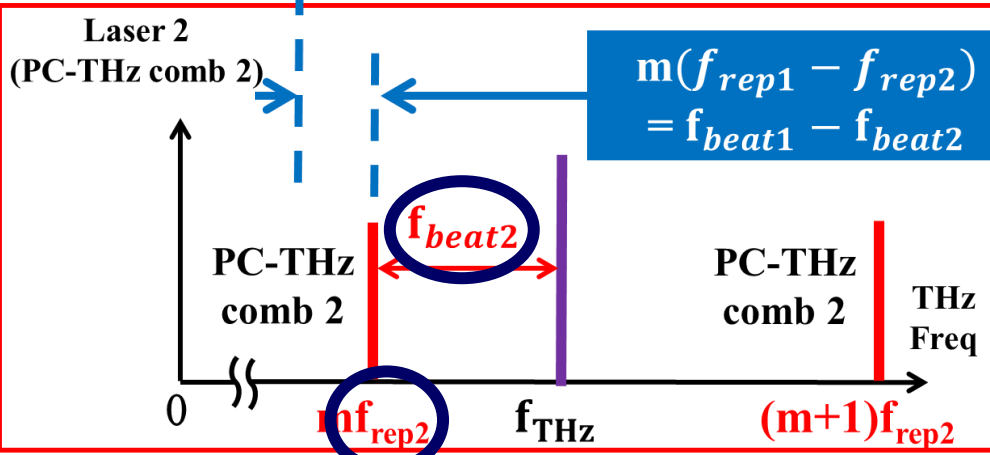
Present talk

Real-time determination of the fast or largely fluctuating CW-THz frequency using dual PC-THz combs .

Real-time determination of CW-THz frequency



parallel measurement using dual PC-THz combs !



$$m = \frac{|f_{beat1} - f_{beat2}|}{|f_{rep1} - f_{rep2}|}$$

$$f_{THz} = mf_{rep1} - f_{beat1} \quad [(f_{beat1} - f_{beat2}) / (f_{rep1} - f_{rep2}) > 0]$$

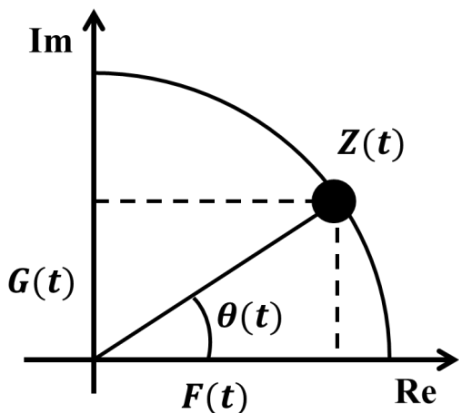
$$f_{THz} = mf_{rep1} + f_{beat1} \quad [(f_{beat1} - f_{beat2}) / (f_{rep1} - f_{rep2}) < 0]$$

Instantaneous frequency measurement using Hilbert transformation

Ref) H. Fuser et al, *Appl. Phys. Lett.* **99**, 121111 (2011).

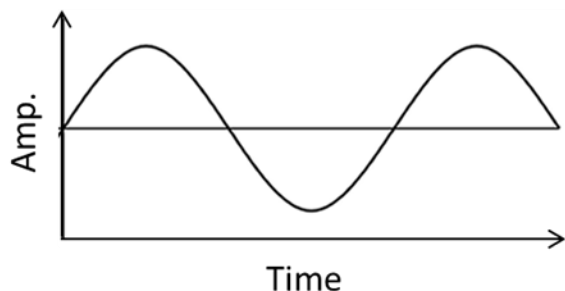
$$Z(t) = F(t) + iG(t)$$

$Z(t)$: analytic signal $F(t)$: measurement signal $G(t)$: signal after Hilbert transform

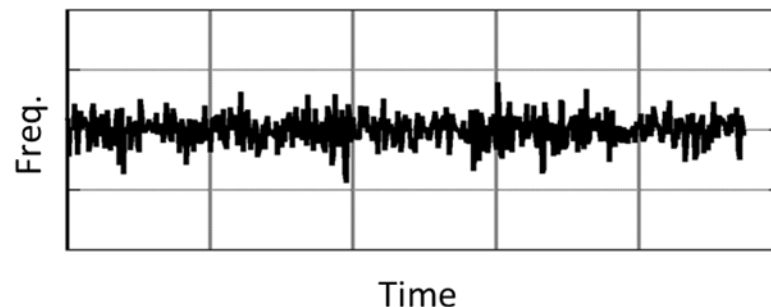


$$\theta(t) = \arg[Z(t)] = \tan^{-1} \left[\frac{G(t)}{F(t)} \right]$$

$$f = \frac{1}{2\pi} \times \frac{d\theta(t)}{dt}$$



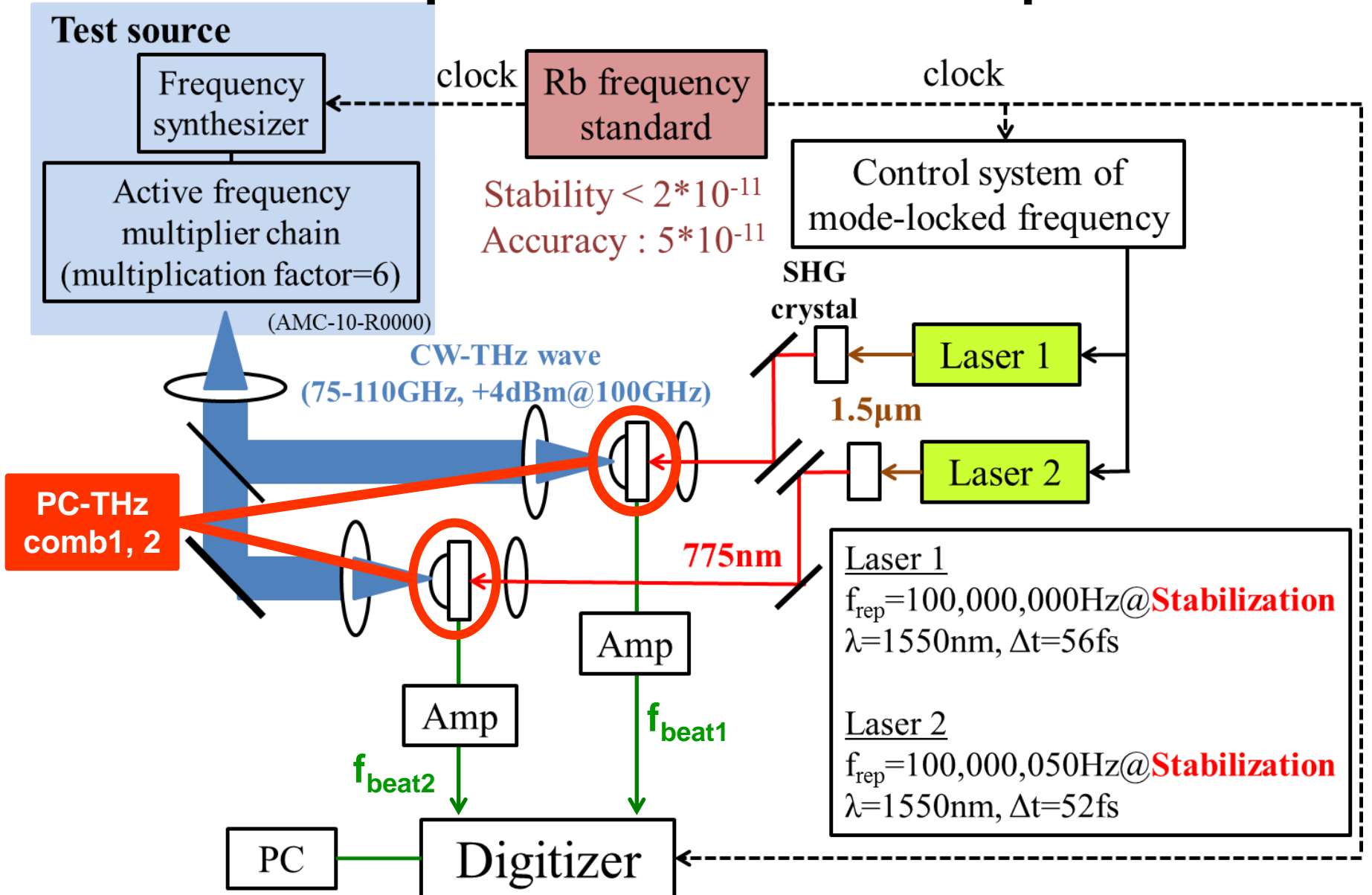
Hilbert transform



Temporal waveform of f_{beat}

Instantaneous frequency of f_{beat}

Experimental setup

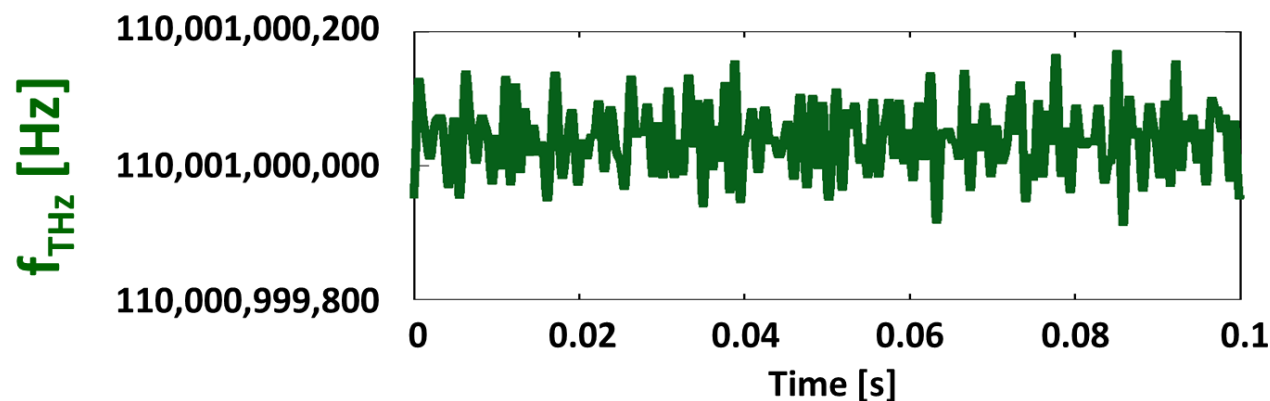
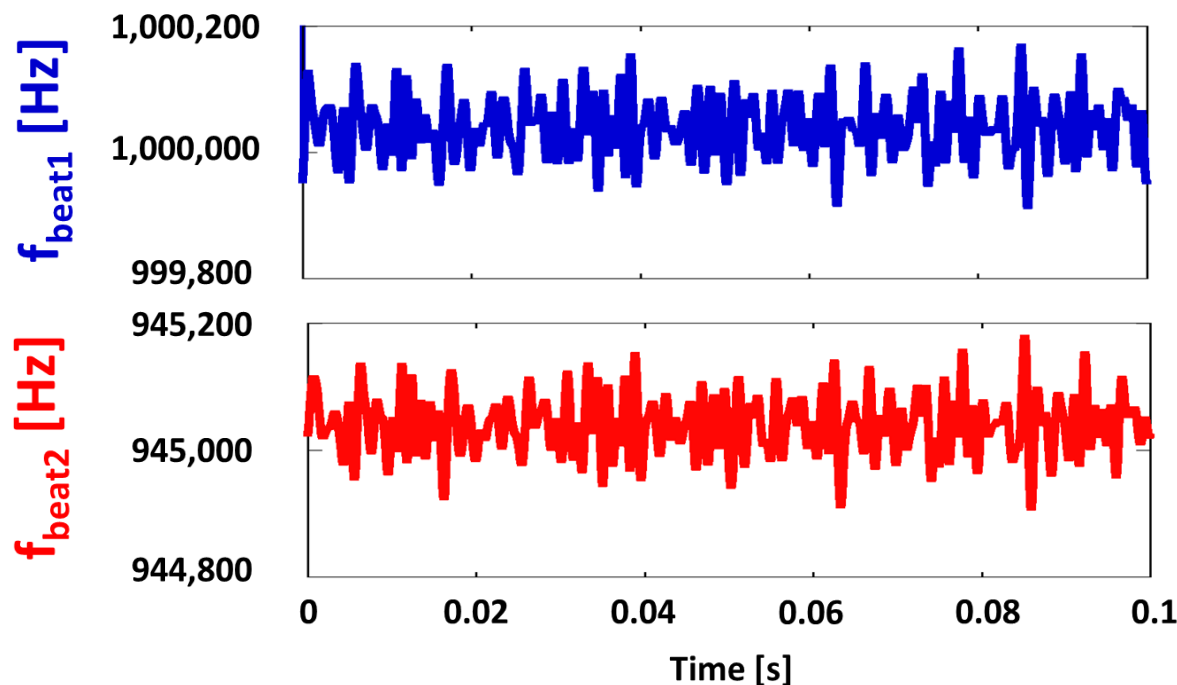


Real-time determination of f_{THz}

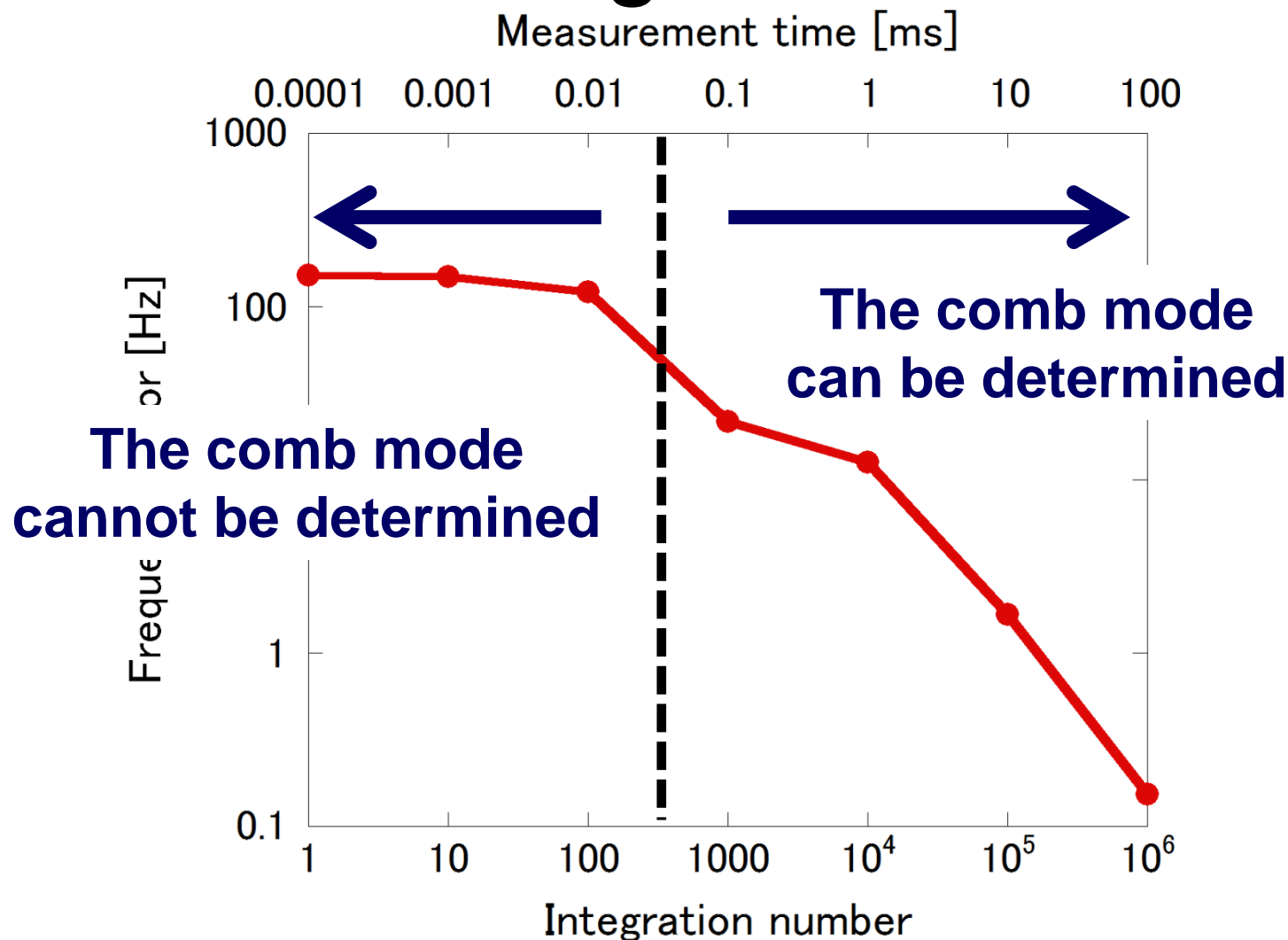
Sampling rate
10MHz

$f_{\text{rep1}} = 100,000,000$ Hz
 $f_{\text{rep2}} = 100,000,050$ Hz

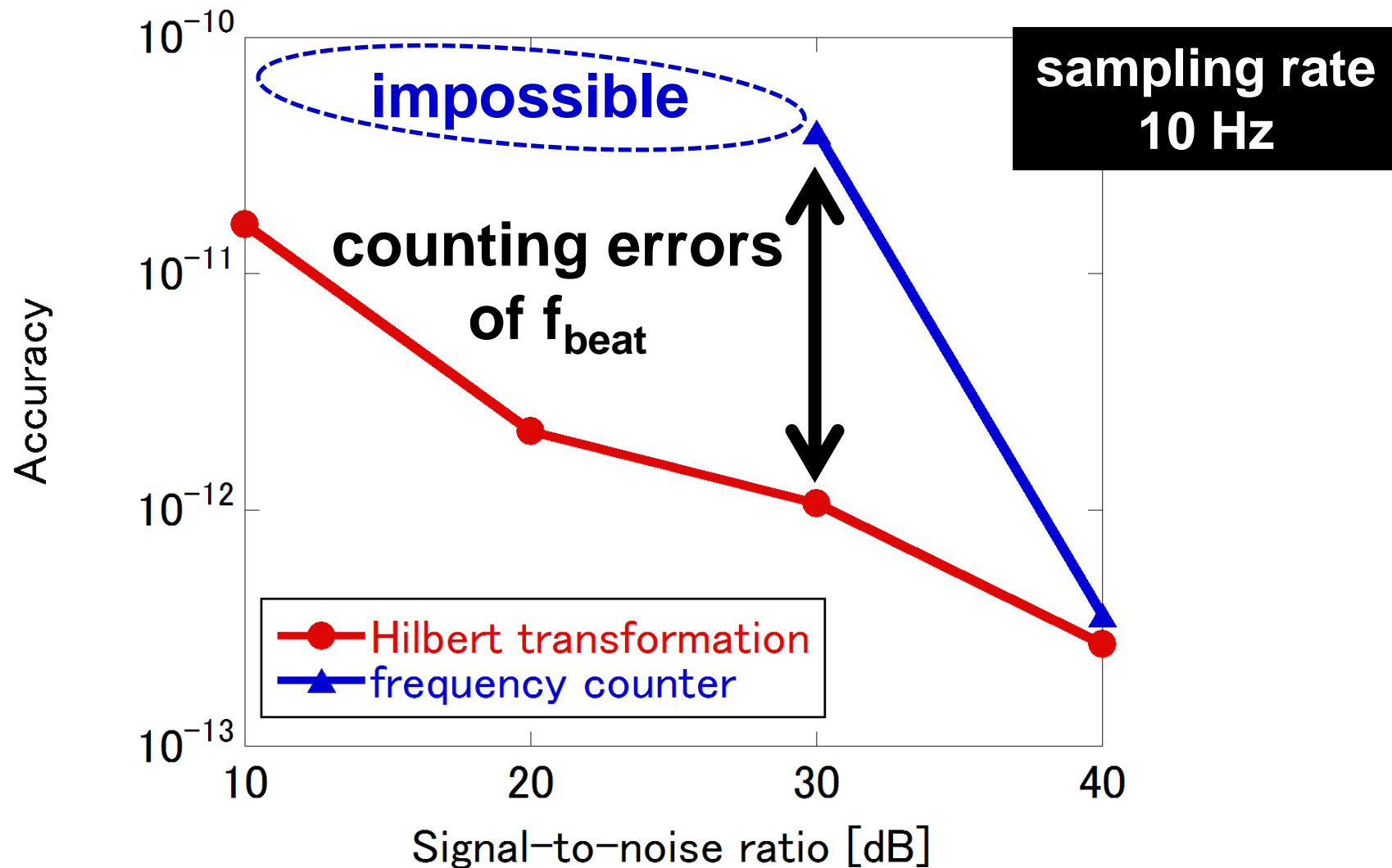
fixed



Frequency error with respect to a various integration number

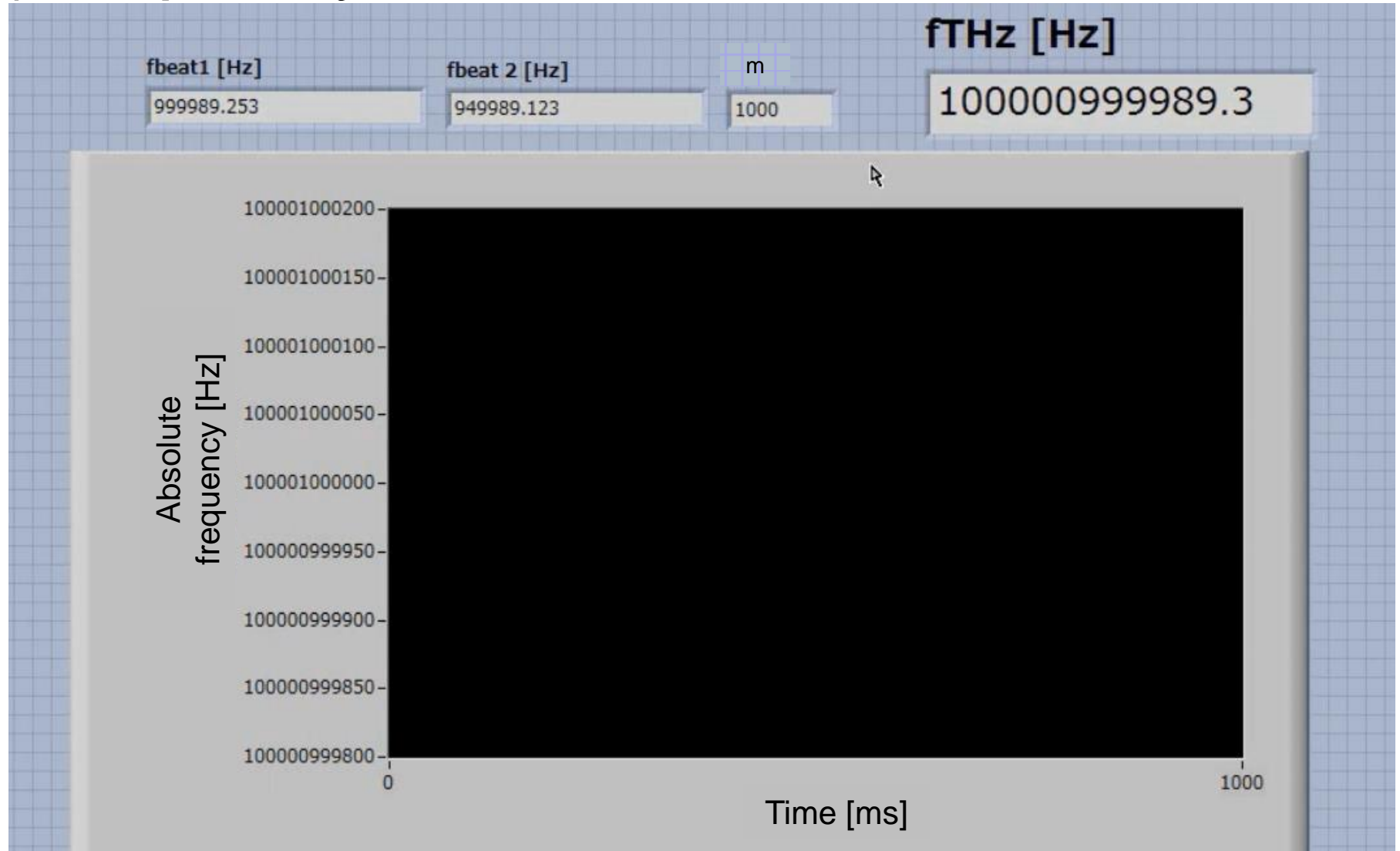


Comparison of accuracy between Hilbert transformation and frequency counter



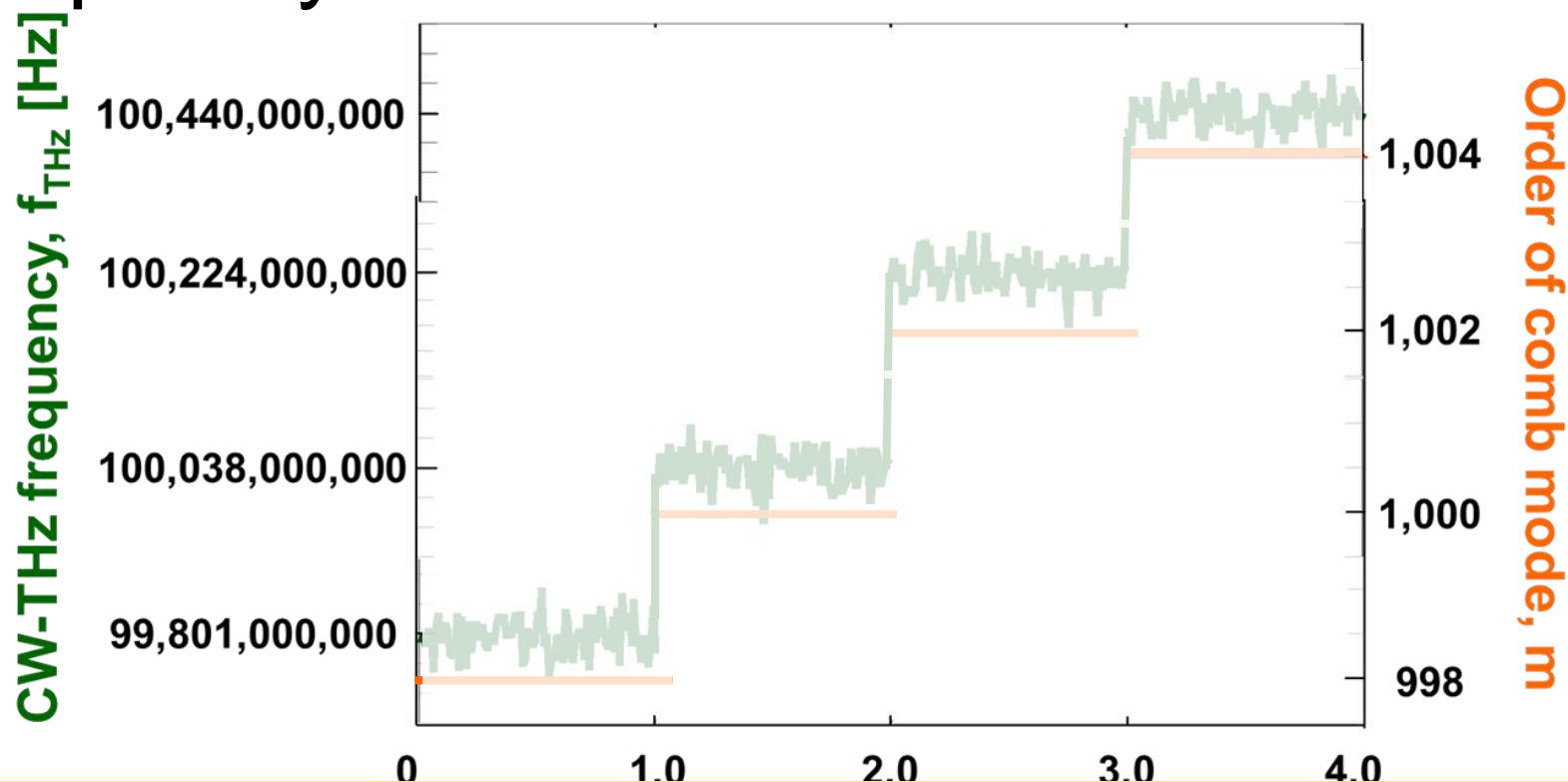
Real-time monitoring of CW-THz wave ①

(Frequency fluctuation = 0.1 THz \pm 100 Hz)



Real-time monitoring of CW-THz wave ②

(Frequency fluctuation = 0.1THz + **200MHz**)



A high potential for real time monitoring of large fluctuation such as mode hopping in CW-THz sources!

Summary

Real-time absolute frequency measurement of the fast or largely fluctuating CW-THz wave using dual PC-THz combs

(1) Frequency accuracy

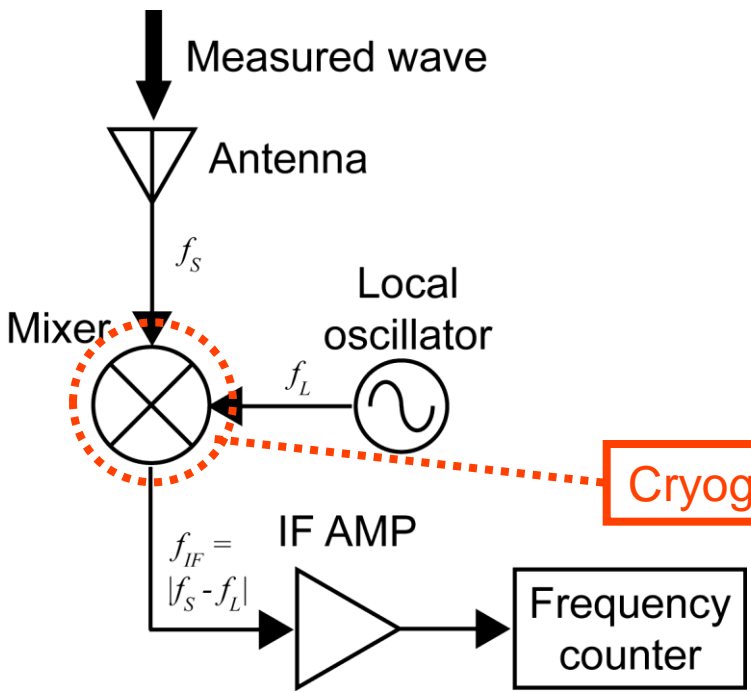
- 2.2×10^{-10} at a sampling rate of 10 kHz
- 1.5×10^{-12} at a sampling rate of 10 Hz

(2) Possible to determine f_{THz} at lower SNR (~ 10)

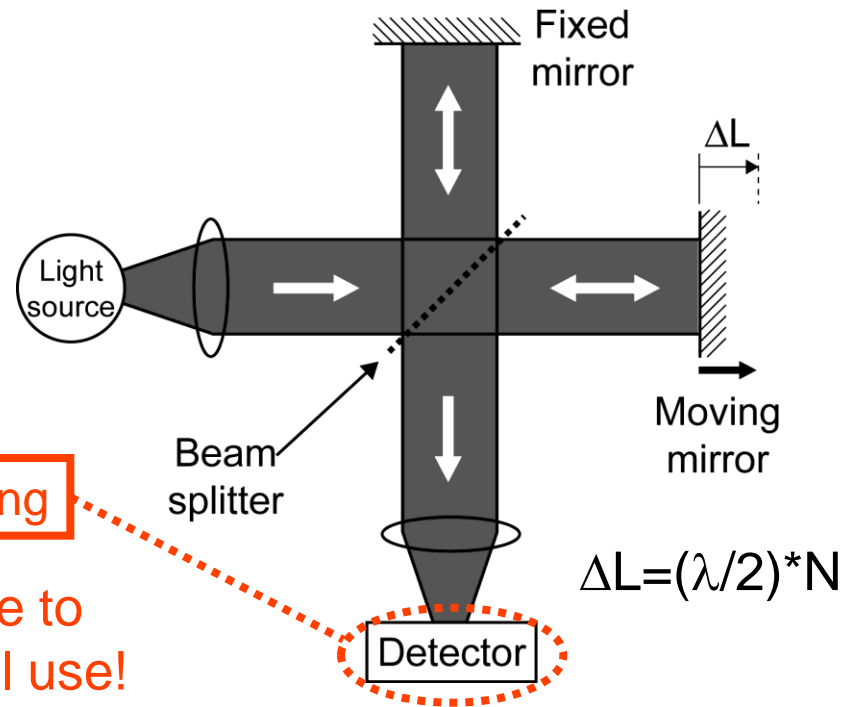
(3) Available for large change of f_{THz} across the comb mode

Conventional method

Electrical heterodyned method



Optical interferometric method



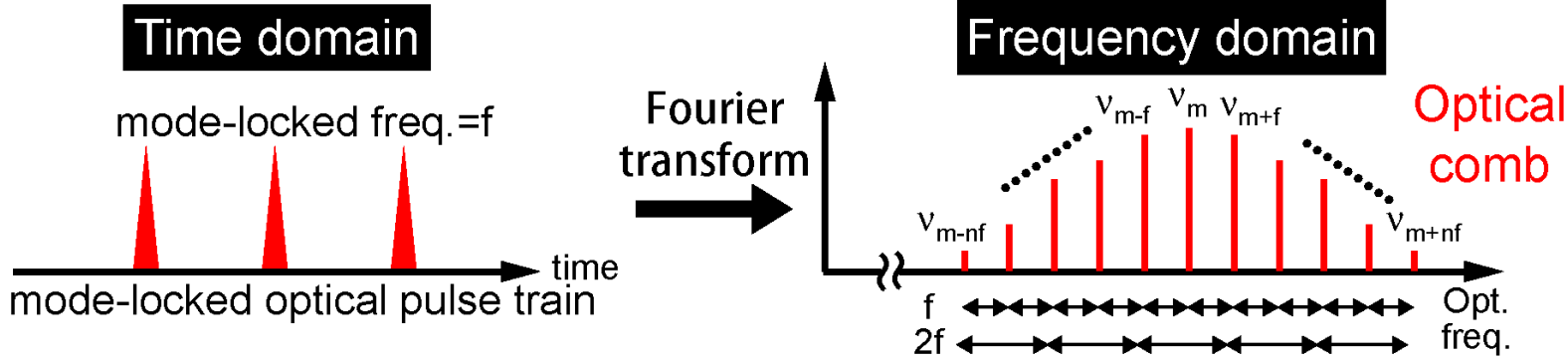
Cryogenic cooling

Obstacle to practical use!

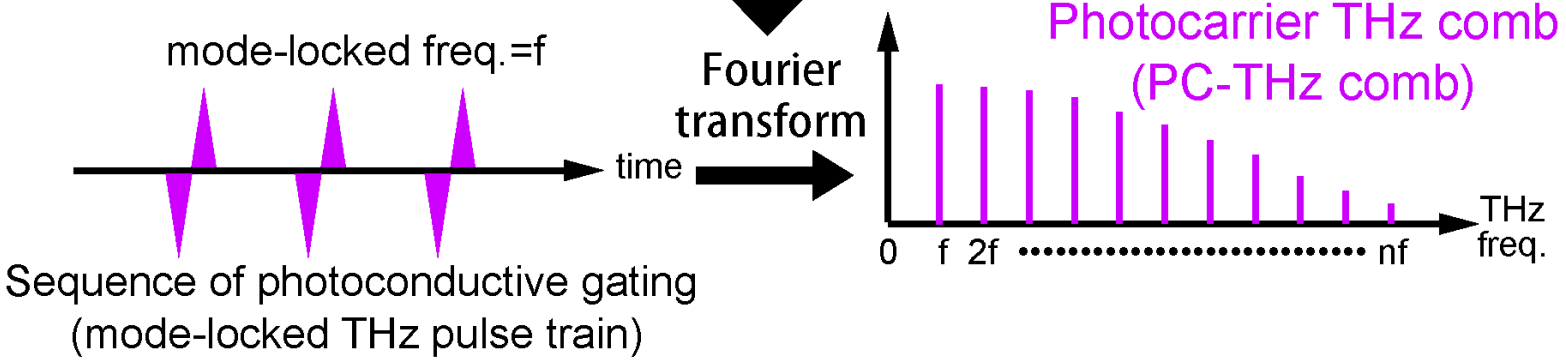
Difficult to cover all frequency region of THz wave (0.1~10THz)

→ Requirement of new method optimized for THz wave!

Optical comb and THz comb



Photoconductive antenna (PCA) for THz detection



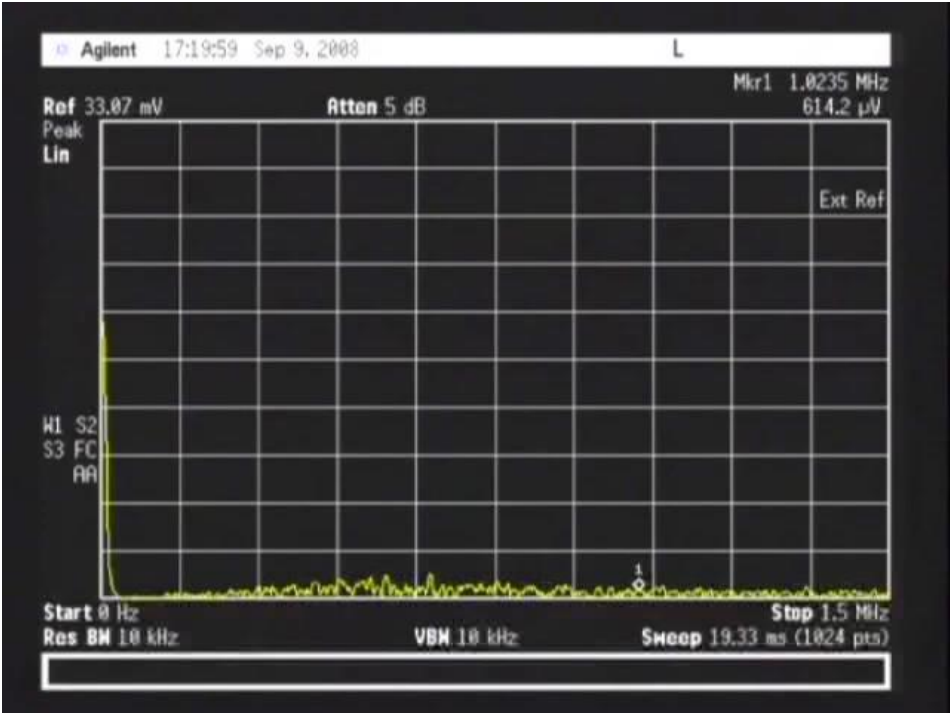
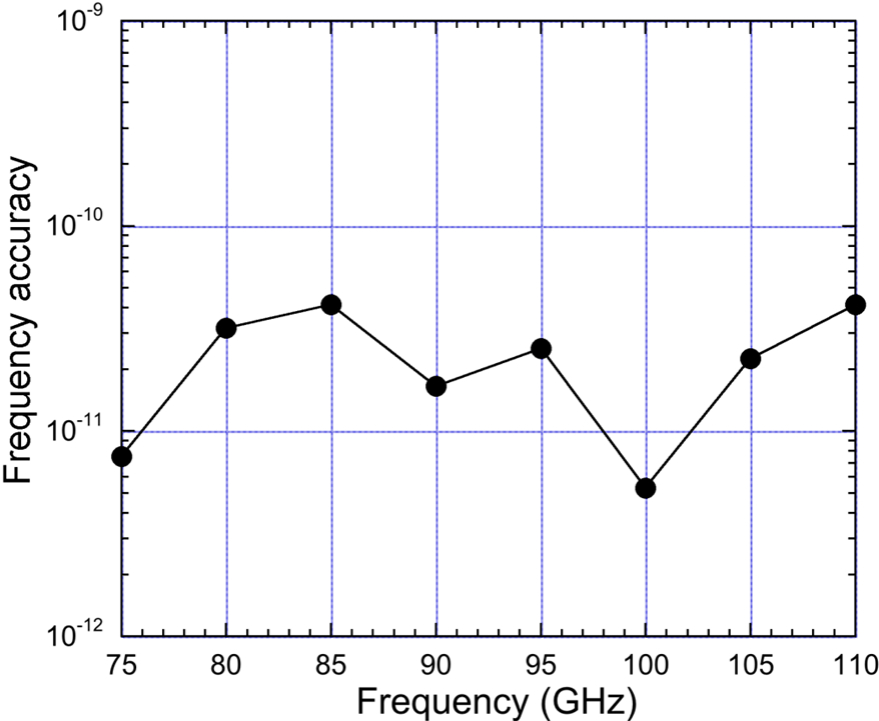
Simple, broadband selectivity, high spectral purity, offset free, and absolute frequency calibration

Previous study

Ref) T. Yasui et al. *Opt. Express* 17, 17034-17043 (2009).

Absolute frequency measurement

Real-time monitoring of CW-THz wave

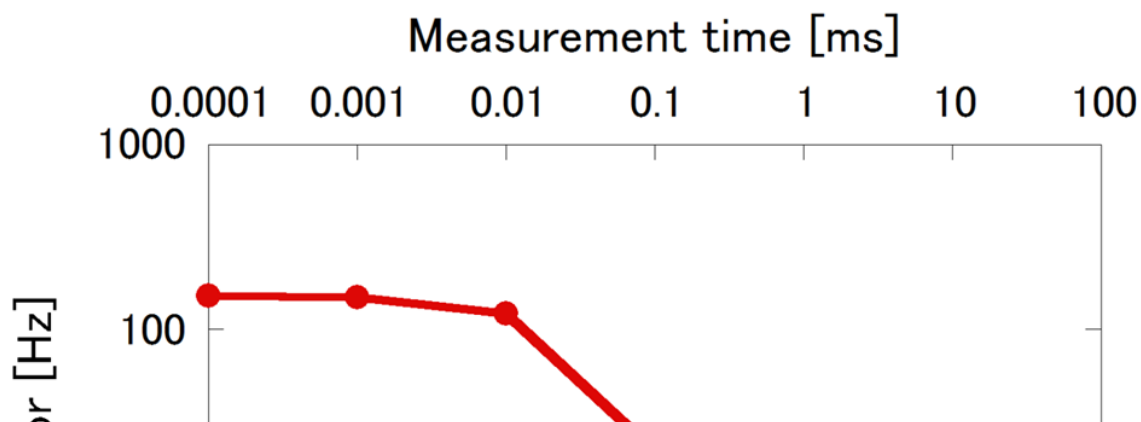


The advantage of Hilbert transformation

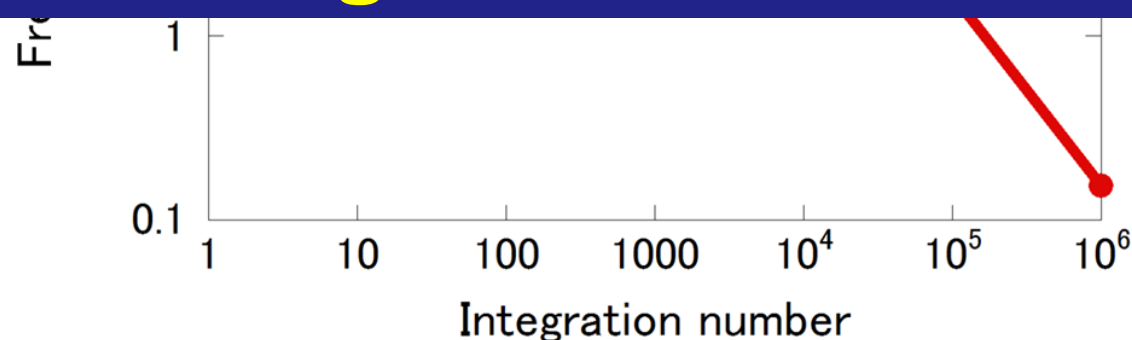
- Compared with frequency counter

1. The beat signal of lower SNR (~ 10) can be measured
2. Fast phenomenon can be measured
→ frequency counter is limited by gate time

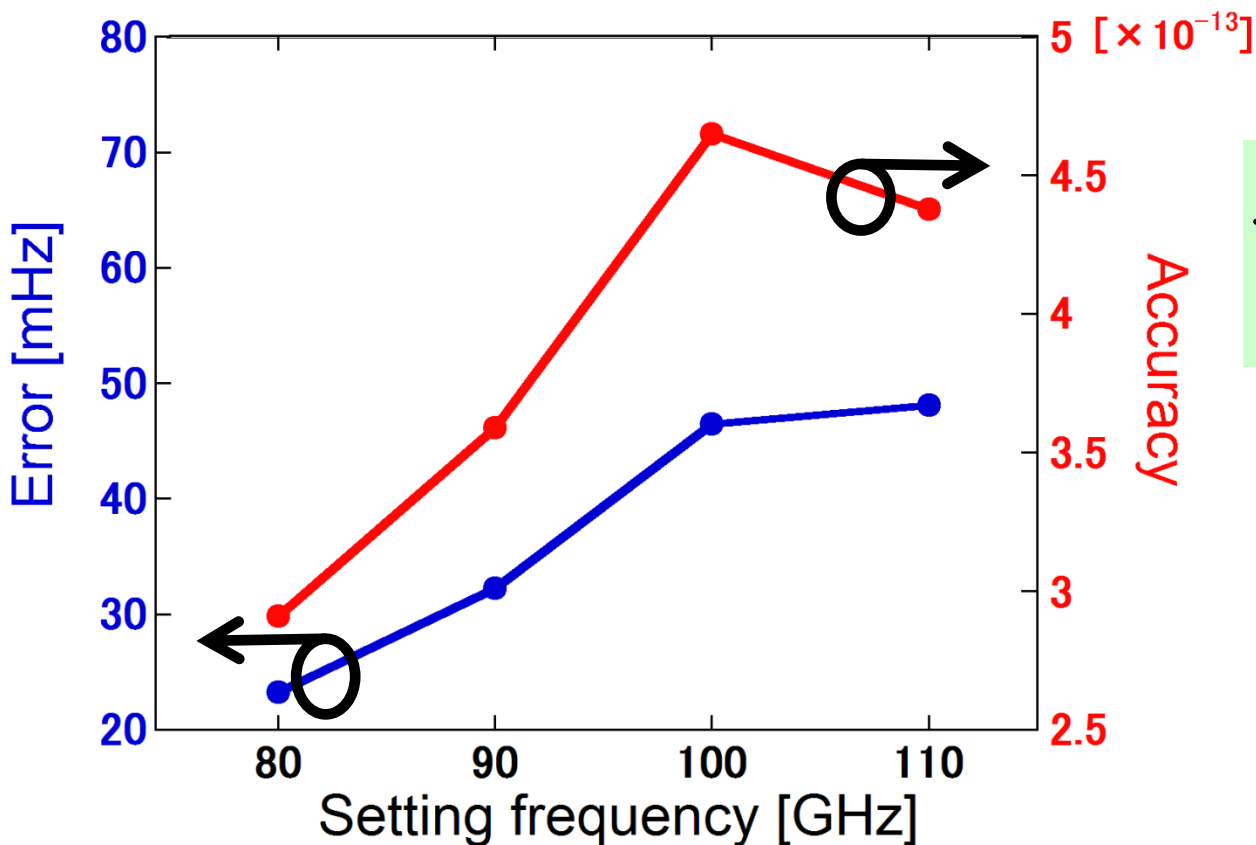
The disadvantage of Hilbert transformation



Trade-off between accuracy and integration number !



Accuracy of absolute frequency measurement



Estimation of measurement error

$$f_{\text{THz}} = m f_{\text{rep1}} + f_{\text{beat1}}$$

$$Df_{\text{THz}} = m Df_{\text{rep1}} + Df_{\text{beat1}}$$

$$\Delta f_{\text{rep1}} = 120 \mu\text{Hz}$$

$$\Delta f_{\text{beat1}} = 21 \text{mHz}$$

$$m = 800 \sim 1100$$

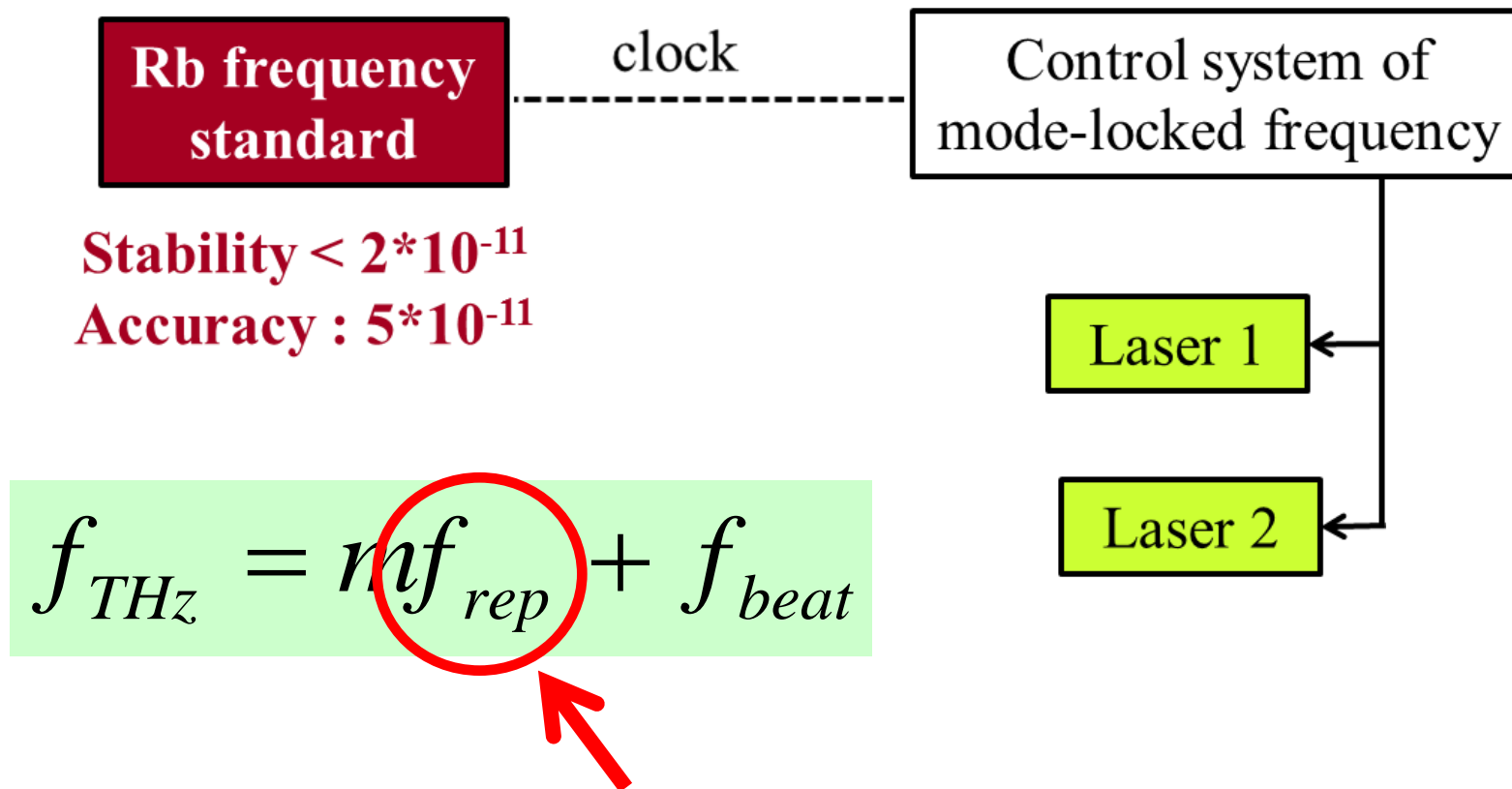
$$\Delta f_{\text{THz}} = 117 \sim 153 \text{mHz}$$

Mean precision = $1.5 \cdot 10^{-12}$

Accuracy

$$\textit{Accuracy} = \frac{\textit{frequency error}}{\textit{setting frequency}}$$

The limitation of accuracy



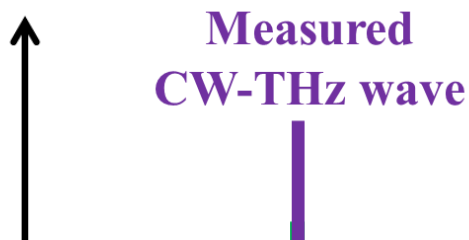
The accuracy is limited by stability of f_{rep}

The fast or largely fluctuation of CW-THz wave

limited by response speed or sensitivity of
PCA detector

However

- $m f_{\text{rep}}$ and f_{THz} are same value $\rightarrow f_{\text{beat}} = 0$



**The m and f_{THz} cannot be determine due to
no beat signal !!**

Real-time absolute frequency measurement using a single PC-THz comb

1. modulating f_{rep}

→ Δf_{rep} and Δf_{beat} can be measured

2. using free-running laser

→ Δf_{rep} and Δf_{beat} can be measured

The m and f_{THz} can be determined by measuring Δf_{rep} and Δf_{beat} at the same time !!