Phase-shifting digital holography

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A new method for three-dimensional image formation is proposed in which the distribution of complex amplitude at a plane is measured by phase-shifting interferometry and then Fresnel transformed by a digital computer. The method can reconstruct an arbitrary cross section of a three-dimensional object with higher image quality and a wider viewing angle than from conventional digital holography using an off-axis configuration. Basic principles and experimental verification are described.

Image formation of three-dimensional (3-D) objects is important for display and optical measurement. Conventional imaging systems that use lenses or mirrors need mechanical focusing that is mechanically complex and time consuming. Holography can record enough information for three-dimensional imaging, but photographic recording and optical reconstruction also require chemical processing and mechanical focusing for the reconstructed image. By digital holography holograms are captured by a video camera and reconstructed by a computer by means of a diffraction integral.[1–5] In this case focusing can be adjusted freely to yield images at arbitrary positions. However, the recording methods reported so far used an off-axis hologram that prohibits effective use of the pixel number of a CCD because of the necessity for carrier fringes. The size of the reconstructed image is also limited by the presence of zero-order and conjugate images. The use of an in-line hologram free from these limitations required digital filtering to suppress the conjugate image.[2] For surface mapping of 3-D objects wavelength-scanning holography with a tunable dye laser was also proposed,[6] but it requires an especially high computation load because of the 3-D Fourier transform including the wavelength axis. Here we propose a new approach to digital holography that is free from these limitations. We measure the complex amplitude of the object wave at the CCD plane located at finite distance in the in-line setup by using phase-shifting interferometry.[7] The phase of the reference waves is changed stepwise, and the resulting four interference fringes are processed by a computer to yield the distribution of the complex amplitude of the wave. Then the distribution is Fresnel transformed in the computer to reconstruct images at arbitrary planes. The principles and experimental results are described below.

We consider the two-beam interferometer shown in Fig. 1. A laser beam is divided into two paths, one of which contains a transmitting or reflecting object and the other, the piezoelectric transducer mirror, which is capable of phase shifting. The phase-shifted interference patterns are taken by a CCD camera and processed by a computer to derive the complex amplitude and image reconstruction. The coordinate systems shown in Fig. 2 are adopted for wave-front analysis and numerical reconstruction. We assume a point object located at (x_0, y_0, z_0). Then the object wave at the hologram plane is represented under a parabolic approximation by

$$U(x, y) = A \exp(i\phi) = \frac{A_0}{z_0} \exp\left[i\phi_0 + ikz_0 + ik\frac{(x - x_0)^2 + (y - y_0)^2}{2z_0} \right], \quad (1)$$

where $A_0 \exp(i\phi_0)$ is the complex amplitude of the point object and k is the wave number. A plane reference wave whose complex amplitude is represented by $U_R(\phi_R) = A_R \exp(i\phi_R)$ is superimposed upon the object wave. The resultant intensity to be recorded by a CCD camera is expressed as

$$I(x, y; \phi_R) = |U_R(\phi_R) + U(x, y)|^2 = {A_R}^2 + A^2 + 2A_R A \cos(\phi_R - \phi). \quad (2)$$

By registering the following intensities with stepped phase differences, we can derive the object phase as

$$\phi(x, y) = \tan^{-1} \frac{I(x, y; 3\pi/2) - I(x, y; \pi/2)}{I(x, y; 0) - I(x, y; \pi)}, \quad (3)$$

where we have assumed the initial reference phase to be zero. The real amplitude $A(x, y)$ of the object wave can also be derived from the intensity that results from blockage of the reference. In a reconstruction by a

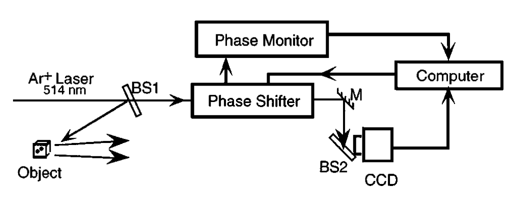

Fig. 1. Arrangement for phase-shifting digital holography: BS1, BS2, beam splitters; M, mirror.

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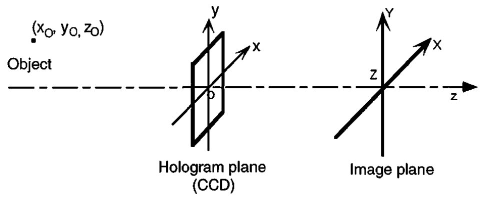

Fig. 2. Coordinate system for hologram recording and reconstruction.

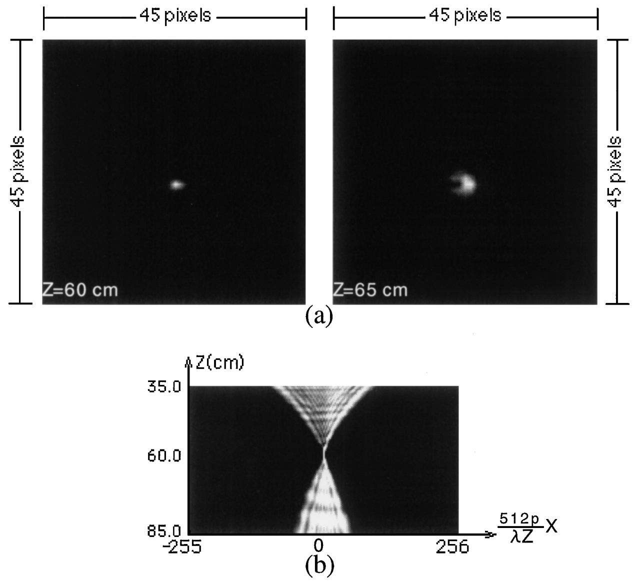

Fig. 3. Reconstructions of (a) a point object located at $x_0 = y_0 = 0$ and $z_0 = -60$ cm and (b) its cross section cut by the plane $Y = 0$, where p is the pixel size (11 μm) of the CCD.

computer we calculate the Fresnel transformation of the object amplitude:

$$U_1(X, Y, Z) = \iint U(x, y) \times \exp\left[ik\frac{(X-x)^2 + (Y-y)^2}{2Z}\right] \mathrm{d}x\mathrm{d}y. \quad (4)$$

After substitution of Eq. (1) we obtain

$$U_1(X, Y, Z) = \frac{A_0}{z_0} \times \exp\left[i\phi_0 + ikz_0 + \frac{ik}{2}\left(\frac{X^2+Y^2}{Z} + \frac{{x_0}^2 + {y_0}^2}{z_0}\right)\right] \times \iint \exp\left\{-ik\left[x\left(\frac{x_0}{z_0} + \frac{X}{Z}\right) + y\left(\frac{y_0}{z_0} + \frac{Y}{Z}\right) - \frac{x^2+y^2}{2}\left(\frac{1}{z_0} + \frac{1}{Z}\right)\right]\right\} \mathrm{d}x\mathrm{d}y, \quad (5)$$

whose intensity becomes maximum at $X = x_0$, $Y = y_0$, $Z = -z_0$, which indicates the formation of the real point image at the original object position. The phase term does not affect the intensity of the image. General objects can be considered an assembly of distributed point sources, and thus Eq. (5) also suggests the reconstruction of a 3-D object.

In experiments an Ar laser of 514.5-nm wavelength and 1-W output power was used. The reference mirror was moved by a piezoelectric transducer actuator controlled by a computer. We controlled the amount of phase shift by constructing a second interferometer whose fringe pattern was taken by a second CCD camera followed by a real-time fringe analyzer that uses an electronic moiré principle and delivers a phase-shift signal.[8] The hologram pattern was taken by a Sony-XC-77 CCD camera with 493 × 768 pixels of 13 μm × 11 μm size. Its output was analog-to-digital converted by 8 bits and stored in a frame grabber with 512 × 512 pixels. For reconstruction by a Sun-4 workstation we performed the integral of Eq. (7) on the basis of a two-dimensional fast-Fourier transform algorithm. Computation time was ~50 s for reconstruction. First we employed a point object formed by a microscope objective (40×; N.A., 0.75) and

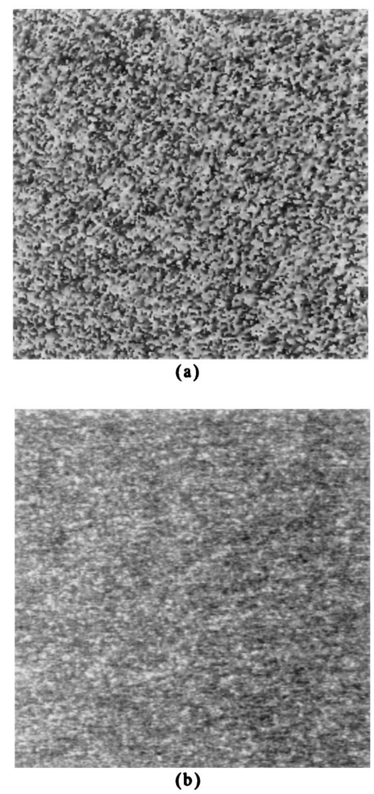

Fig. 4. (a) Phase and (b) amplitude maps of a diffusely reflecting object.

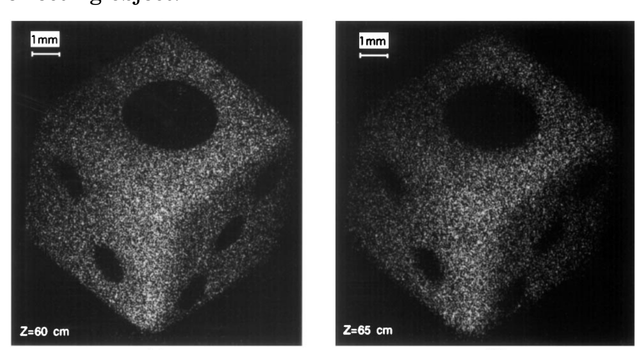

Fig. 5. Numerically reconstructed images of a diffusely reflecting object: (a) focused image, (b) defocused image.

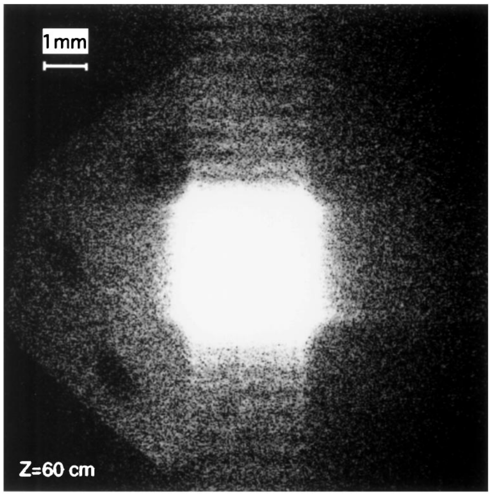

Fig. 6. Reconstructed image from a single interference pattern corresponding to a Gabor hologram.

located at $z_0 = -60$ cm. Figure 3(a) shows the reconstructed images at $Z = 60$ and $Z = 65$ cm. The cross section of the image cut by the $X-Z$ plane is shown in Fig. 3(b). The half-value width of the focused image is 2.5 pixels. As a 3-D diffuse object we chose a die as large as 7 mm $\times$ 7 mm $\times$ 7 mm whose center was positioned at a distance of $z_0 = 60$ cm. The laser power was 100 mW. Figure 4 shows phase and amplitude maps that result from this algorithm. The images reconstructed at $Z = 60$ cm and $Z = 65$ cm are shown in Fig. 5. We can clearly observe the 3-D effect in the images. For comparison, an image reconstructed from a single interference pattern that corresponds to a Gabor hologram is shown in Fig. 6. The desired image is strongly disturbed by the zero-order and the conjugate images. We also performed reconstruction only from the phase map, as happens in reconstruction from a kinoform.[9] The resultant image was not much different from Fig. 5.

Now we discuss the general imaging quality of the present system. The angular size α of the object to be reconstructed clearly is given by the resolution p of the CCD camera such that $\alpha = \lambda/p$ because the finest fringe spacing formed between the light emanating from the opposite edges of the object is given by $\lambda/2\,\sin(\alpha/2) \cong \lambda/\alpha$. α becomes a few tenths of radian. The resolution cell in the reconstructed image depends on the number of CCD pixels because the highest spatial frequency in the reconstructed image is given by $\xi = Np/\lambda z_0$ with the pixel number N along the CCD line. Hence the space–bandwidth product of digital holography is simply given by

$$P = \alpha z_0 \xi = a/p = N\,. \tag{6}$$

In two-dimensional space the product becomes N^2 if we assume a square CCD. Thus the imaging capacity of the present digital holography is the same as that for conventional imaging. In the reconstruction the term neglected in the Fresnel transformation might also be useful to improve the image quality. By using a point reference we could also reconstruct magnified or reduced images.

A new method has been developed that realizes three-dimensional imaging and produces high image quality without imaging lenses. It uses the diffraction transform of the complex amplitude at a CCD plane, detected by a phase-shifting interferometry. Electronic focusing provides high flexibility in display of 3-D objects. Although the computation load is heavy for the large number of pixels needed for high-quality imaging, this difficulty could be overcome by use of a parallel array processor for Fresnel transformation. The new method will be especially useful for quantitative measurement of shape or deformation of 3-D surfaces because all the essential information of the objects is acquired as input data and subsequent processing is accomplished by a computer until the desired quantities are extracted and displayed. The technique can also be applied to waves for which no imaging component is available, such as ultraviolet and X rays, if coherent sources and phase-shifting interferometry can be used. It might be also used to transmit 3-D image information through ordinary communication channels.

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