Terahertz spectrum analyzer based on a terahertz frequency comb

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Abstract: Precision frequency measurements of terahertz (THz) waves are required to establish metrology in the THz spectral region. However, frequency measurement techniques in this region are immature. We propose a THz spectrum analyzer to measure the absolute frequency and spectral shape of continuous-wave THz waves. Based on a stable frequency comb generated in a photoconductive antenna, the absolute frequency of a sub-THz test source was determined at a precision of 2.8\times10\textsuperscript{-11}. Furthermore, the spectral bandwidth of the THz spectrum analyzer can be extended over 1 THz, as demonstrated by measurement of a THz test source. This spectrum analyzer has the potential to become a powerful tool for THz frequency metrology.

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1. Introduction

Frequency is a fundamental physical quantity of electromagnetic wave. Therefore, various techniques have been established to measure the frequency of electromagnetic waves. Interferometry is often used in the optical region. For precise measurements, the number of interference fringes of a measured monochromatic source is compared with that of a standard signal having a known optical frequency (e.g., a frequency stabilized laser) over a given path length. On the other hand, a heterodyne method is typically used at microwave and millimeter wavelengths. In this method, a measured wave is detected with an antenna and then electrically mixed with a standard signal from a local oscillator of known frequency, resulting in the generation of a heterodyne beat signal at radio frequencies (RF). The beat frequency can be precisely measured with RF instruments.

In the past decade, terahertz (THz) radiation, lying at the boundary between optical and electrical waves, has emerged as a new mode for sensing and communication. THz spectral fingerprints are useful in various sensing applications [1]. In addition, THz waves are expected to find application as a carrier wave for broadband wireless communications [2]. Following an increase of such THz applications, a need for THz frequency metrology has arisen. Furthermore, the advent of practical continuous-wave THz (CW-THz) sources, such as THz quantum cascade lasers [3], uni-traveling-carrier photodiodes [4], and resonant tunneling diodes [5], have increased demand for precise frequency measurement of CW-THz waves. However, it has proved difficult to cover the full range of THz radiation (typically, 0.1 to 10 THz) because existing methods are not optimized for the THz region. Furthermore, existing instruments often require cooling to suppress thermal noise. Recently, new techniques for the frequency measurement of CW-THz waves have been proposed. A non-synchronized electro-optic sampling technique using a mode-locked Ti:sapphire laser has been applied to the frequency measurement of a CW CO\textsubscript{2} laser at 28 THz [6]. The frequency measurement of a 50-GHz CW wave from a Gunn oscillator was performed by combining use of a photoconductive antenna (PCA) for a heterodyne receiver with the optical intermode beat of a laser diode for a local oscillator [7]. However, the signal-to-noise ratio is low in the former method, and it is difficult to achieve precise measurements of the absolute frequency in either method.

In the optical region, optical frequency combs have emerged as a new mode for frequency metrology [8]. Since the frequency comb provides attractive features for frequency metrology, namely excellent accuracy and stability, broadband selectivity, ultra-narrow linewidth, and exact multiplication, it can be used as a precise ruler in the frequency domain. Recently, we have extended the concept of a frequency comb to the THz region using multi-frequency-heterodyning photoconductive detection [9]. Since this so-called THz comb is a harmonic comb of the laser mode-locked frequency without any offset, it enables absolute frequency measurements using the mode-locked frequency and order of the comb mode. Such a THz comb is grouped into two modes: an electromagnetic THz comb (EM-THz comb) and a photocarrier THz comb (PC-THz comb). When a PCA is used as a THz emitter by adding an electrical bias between electrodes of the antenna, an EM-THz comb is radiated from the PCA and propagates in free space. Conversely, when one uses a PCA connected to an ammeter as a THz detector, a sequence of instantaneous photoconductive gates by the probe pulse light induces a PC-THz comb in the PCA. It can then be used as a THz detector equipped with a ruler for the absolute frequency because the PCA and the PC-THz comb respectively act as a heterodyne receiver and a local oscillator with multiple frequencies. Previously we measured the detailed frequency spectrum of an EM-THz comb using a PCA having a PC-THz comb [9]. This concept can be applied to frequency measurements of CW-THz waves. In this paper, we propose a new spectrum analyzer to measure the absolute frequency and spectral shape of CW-THz waves. We demonstrate its application to determine the absolute frequency of a CW-THz wave radiated from a test source.
2. Principle of operation

In the field of microwave technology, harmonic mixing technique based on electro-optic sampling has been often used to down-convert microwave spectrum to RF spectrum using the comb spectrum of the mode-locked laser with a nonlinear detection technique [10, 11]. We here modified this technique to down-convert THz spectrum to RF spectrum using photoconductive detection. Our THz spectrum analyzer is based on a heterodyne technique as shown in Fig. 1. Compared with the electrical heterodyne method, a key difference here is that the PCA is used as both a detector and a mixer for the measured CW-THz wave. Use of a PCA as a heterodyne receiver achieves high, broadband spectral sensitivity in the THz region without the need for cooling. Another difference is the use of a PC-THz comb as a local oscillator with multiple frequencies covering from the sub-THz to the THz region.

Consider a PCA detector when an fs-ML laser light (probe pulse with mode-locked frequency $f$) is incident on the antenna gap of a photoconductive film, as shown in Fig. 2(a). Figure 2(b) illustrates the corresponding spectral behaviors in optical, THz, and RF regions. The probe pulse train emitted from the fs-ML laser constructs an optical frequency comb in the frequency domain, whose mode spacing is equal to a mode-locked frequency [upper part of Fig. 2(b)]. When the PCA is triggered by such pulse train, time-varying photo-created carriers are generated in the PCA due to the quadratic detection which squares the electric field of the input light and then converts the optical intensity to electric carriers. Resultant photo-created carriers function as a sequence of instantaneous photoconductive gating. Defining $N(t)$ as the number of photo-created carriers, the Fourier transform of $N(t)$, $N(\omega)$, also forms a comb structure, i.e., PC-THz comb [middle part of Fig. 2(b)]. Because the generation of the PC-THz comb can be considered as an ultra-wideband demodulation of an optical frequency comb via the PCA, the optical comb (frequency spacing of $f$) is down-converted to the THz region without any change in its frequency spacing. The resulting PC-THz comb is a harmonic frequency comb without any offset, composed of a fundamental component (of frequency $f$) and a series of harmonic components (at $2f$, $3f$, ..., $nf$) of a mode-locked frequency. This is the biggest difference compared to an optical frequency comb having a carrier-envelope offset frequency, and it enables us to achieve simple, practical THz frequency metrology including stabilization of a THz comb.

Next, consider what happens when a measured CW-THz wave [$E_{THz}(t)$ in the time domain or $E_{THz}(\omega)$ in the frequency domain] is incident on a PCA detector triggered by the probe pulse train. The photo-created carriers, $N(t)$, are accelerated by $E_{THz}(t)$ and this acceleration of $N(t)$ is detected as a transient photocurrent, $J(t)$. Thus, $J(t)$ is proportional to the product of $N(t)$ and $E_{THz}(t)$. This product relationship in the time domain leads to the convolution relationship in frequency domain. Therefore, the Fourier transform of $J(t)$, $J(\omega)$, from the PCA is given as the convolution of the CW-THz wave $E_{THz}(\omega)$ and the PC-THz comb $N(\omega)$ as
shown in lower part of Fig. 2(b), which is defined as photoconductive mixing. Such a photoconductive mixing process in the PCA generates a group of beat signals between the CW-THz wave and the PC-THz comb in the RF region. Focus on a beat signal at the lowest frequency. Since the beat signal (of frequency \(f_b\)) is generated by mixing the CW-THz wave (of frequency \(f_x\)) with the \(m\)-th comb mode (of frequency \(mf\)) nearest in frequency to the CW-THz wave, the \(f_b\) value is given as follows

\[
f_b = |f_x - mf|. \quad (1)
\]

Therefore, if the order of \(m\) and the sign of \(f_x - mf\) are measured, the \(f_x\) value can be determined. In order to determine the order of \(m\) and the sign of \(f_x - mf\), the mode-locked frequency is changed from \(f\) to \(f + \delta f\) by adjustment of the laser cavity length. This results in a change of the beat frequency to \(f_b + \delta f_b\). Since \(|\delta f_b|\) is equal to \(|m\delta f|\), the order of \(m\) is determined as

\[
m = \frac{\delta f_b}{\delta f}. \quad (2)
\]

The sign of \(\delta f_b/\delta f\) equals inverse of that of \(f_x - mf\). Finally, the absolute frequency of the measured CW-THz wave can be determined by measuring \(f, f_x, \delta f,\) and \(\delta f_b\) because

\[
f_x = mf + f_b \quad (\delta f_b/\delta f < 0) \quad (3a)
\]

and

\[
f_x = mf - f_b \quad (\delta f_b/\delta f > 0). \quad (3b)
\]

![Fig. 2. (a). Geometry of a measured CW-THz wave and probe pulse light on the PCA, and (b) the corresponding spectral behavior in optical, THz, and RF regions.](image)

### 3. Experimental setup

Figure 3 illustrates the setup of our THz spectrum analyzer. A Kerr-lens mode-locked Ti:sapphire laser (FEMTOLASERS Femtosource Scientific Pro with pulse energy = 3.7 nJ, pulse duration = 10 fs, central wavelength = 780 nm, and repetition rate = 81.8 MHz) was employed as a probe laser to generate a PC-THz comb in a PCA. The mode-locked frequency \(f\) was stabilized using a laser control system referred to a rubidium frequency standard (Stanford Research Systems FS725 with frequency = 10 MHz, accuracy = 5×10\(^{-11}\), stability = 2×10\(^{-11}\) at 1 s) [9]. Because the frequency fluctuation between \(f\) and the rubidium frequency standard corresponds to the instability of 2.7×10\(^{-14}\) (1 sec), the resultant instability and accuracy of \(f\) were regarded as equivalent to those of the rubidium frequency standard. Such
the accuracy and instability of $f$ enable us to generate a PC-THz comb in the PCA equivalent to the rubidium frequency standard.

A probe light (with an average power of 10 mW) from the stabilized Ti:Sapphire laser is focused onto an antenna gap in a PCA using a lens, resulting in the generation of a PC-THz comb. Although the PC-THz comb is generated consecutively from the RF to the THz region without gap, the actual spectral sensitivity of the THz spectrum analyzer is limited by the antenna shape of the PCA and/or the phonon absorption in the PCA materials. We used two kinds of the PCA: a bowtie-shaped, low-temperature-grown GaAs (LT-GaAs) PCA (bowtie length = 1 mm and gap = 5 µm) having peak sensitivity around a few hundred GHz for measurement of a sub-THz test source and a dipole-shaped, LT-GaAs PCA (length = 7.5 µm, width = 10 µm, and gap = 5 µm) having high sensitivity over 1 THz for measurement of a THz test source. A CW-THz wave from a test source is incident on the PCA through a spherical silicon lens from the opposite direction of the probe light, without any collimating or focusing of the CW-THz beam. Photoconductive mixing between the CW-THz wave and the PC-THz comb generates a current signal from the PCA. The output current is amplified by a wideband video amplifier (bandwidth = 900 MHz and gain = 30 dB) or a high-gain current preamplifier (bandwidth = 1 MHz and sensitivity = $4 \times 10^6$ V/A) and then measured with an RF spectrum analyzer (Agilent E4402B with a frequency range of 100 Hz to 26.5 GHz) to determine its frequency and spectral shape. The mode-locked frequency $f$ is measured using an RF frequency counter (Agilent 53132A with a frequency range to 225 MHz). The spectrum analyzer and frequency counter are locked to the rubidium frequency standard used for the laser stabilization control.

**Fig. 3.** Experimental setup of THz spectrum analyzer for measurement of a sub-THz test source. fs-ML Ti:S laser: femtosecond mode-locked Ti:sapphire laser; PZT: piezoelectric transducer; BS: beam splitter; L: objective lens; PCA: photoconductive antenna; Si-L: silicon lens; PD: photodetector; AMP: amplifier.

### 4. Results

#### 4.1 Measurement of a sub-THz test source

To test the THz spectrum analyzer, an active frequency multiplier chain (Millitech AMC-10-R0000 with multiplication factor = 6, output frequency = 75–110 GHz, and output power = 4 dBm = 2.5 mW) driven by a frequency synthesizer (Agilent E8257D with frequency = 12.5–18.33 GHz, power = 10 dBm, and linewidth < 0.1 Hz) is used as a test source in the sub-THz region. Since the frequency synthesizer is synchronized to the rubidium standard used for control of the mode-locked frequency, any frequency fluctuations arising from the standard is cancelled as the common-mode fluctuations in this demonstration. The current signal from the PCA passes through the wideband video amplifier and is then measured by the spectrum analyzer. Figure 4(a) shows the resulting signal spectrum when the output frequency of the test source and the laser mode-locked frequency are set at 90,008,480,000 Hz and 81,840,000 Hz.
Hz, respectively [sweep time = 100 ms and resolution bandwidth (RBW) = 300 kHz]. Many beat signals between the CW-THz wave and the PC-THz comb were observed. Figure 4(b) shows an expansion of the spectral region from 0 to 100 MHz with a sweep time of 100 ms and an RBW of 300 kHz. One sees a pair of beat signals at $f_b$ and $f-f_b$. The $f_b$ signal at 23.6 MHz is a beat signal between the CW-THz wave and the $m$-th comb mode nearest in frequency to the CW-THz wave, while the $f-f_b$ signal at 58.2 MHz is a beat signal between the CW-THz wave and the $(m+1)$th or $(m-1)$th comb mode. To further expand the spectrum of the $f_b$ signal, the beat frequency was set below 1 MHz by adjusting the output frequency of the test source and the mode-locked frequency because the bandwidth of the employed high-gain current preamplifier is 1 MHz. Linear-scale and logarithmic-scale spectra of the $f_b$ beat signal were then measured by the RF spectrum analyzer as shown in Figs. 4(c) and 4(d) with a sweep time of 2.3 s and an RBW of 1 Hz. The resulting spectral linewidth was 1.35 Hz. On the other hand, the signal-to-noise ratio (SNR) of the $f_b$ beat signal in Fig. 4(d) was 50 dB. From this SNR and the output power of the test source (namely +4 dBm = 2.5 mW), the detection limit of the THz power is estimated to -46 dBm (or 25 nW) for a sweep time of 2.3 sec.

To determine the absolute frequency of a test source, it is necessary to measure the deviation of the beat frequency while tuning the mode-locked frequency. The initial mode-locked frequency is set at 81,823,757.000 Hz and then the beat signal at the lowest frequency is measured by combining the high-gain current preamplifier and the RF spectrum analyzer. The resulting spectrum of the beat signal is plotted by the red curve in Fig. 5 (sweep time = 773 ms and RBW = 1 kHz). The beat frequency at $f = 81,823,757.000$ Hz was determined to
be 454,027.976 Hz by the spectrum analyzer. Next, the mode-locked frequency was increased to 81,823,857.000 Hz by changing the reference frequency of the stabilized control of the mode-locked frequency. This results in a deviation of the beat frequency indicated by the blue curve in Fig. 5. The beat frequency then became 333,027.731 Hz. By substituting these values into Eq. (2), one obtains

$$m = \frac{333,027.731 - 454,027.976}{81,823,857 - 81,823,757} = 1210.00245$$

and

$$\frac{\delta f_b}{\delta f} < 0.$$  

Therefore, the absolute frequency $f_x$ of the test source was determined using Eq. (3a) to be

$$f_x = mf + f_b = 1210 \times 81,823,757 + 454,027.976 = 99,007,119,997.976 \text{ Hz.}$$  

Since the actual frequency of the test source was 99,007,200,000.000 Hz, the error between the set and measured frequencies was only 2.024 Hz.

Next, we determined the absolute frequency of the test source while tuning its output frequency from 75 to 110 GHz at 5-GHz intervals. Table 1 summarizes the results of setting $f_x$, $f$, $\delta f$, $f_b$, $\delta f_b$, $m$, measured $f_x$, error between setting and measured $f_x$, and precision at each output frequency. The positive frequency offset was measured as an error at each frequency. These offset errors are likely due to systematic errors caused by the frequency synthesizer and/or spectrum analyzer used in the present setup, due for example to insufficient frequency calibration of the instruments. When the measurement precision of the THz spectrum analyzer is defined as a ratio of the error to the set value, a mean precision of $2.8 \times 10^{-11}$ was achieved in this demonstration.
Table 1. Results of absolute frequency measurement of the sub-THz test source when tuning its output frequency from 75 to 110 GHz at 5-GHz intervals.

<table>
<thead>
<tr>
<th>Setting (f_x) (Hz)</th>
<th>(f) (Hz)</th>
<th>(f_b) (Hz)</th>
<th>(\delta f) (Hz)</th>
<th>(\delta f_b) (Hz)</th>
<th>Measured (f_x) (Hz)</th>
<th>Error (Hz)</th>
<th>Precision</th>
</tr>
</thead>
<tbody>
<tr>
<td>75.033,334,800</td>
<td>359,996.</td>
<td>637</td>
<td>37</td>
<td>3.36</td>
<td>4.482×10^{-11}</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>80.024,623,200</td>
<td>359,999.</td>
<td>487</td>
<td>8</td>
<td>0.51</td>
<td>6.411×10^{-11}</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>85.015,851,600</td>
<td>299,995.</td>
<td>103,898.97</td>
<td>34</td>
<td>4.36</td>
<td>5.136×10^{-11}</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>90.007,140,000</td>
<td>299,997.</td>
<td>110,000.39</td>
<td>34</td>
<td>2.34</td>
<td>2.602×10^{-11}</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>94,998,446,400</td>
<td>317,997.</td>
<td>116,099.85</td>
<td>85</td>
<td>2.31</td>
<td>2.437×10^{-11}</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>99,989,728,800</td>
<td>311,997.</td>
<td>122,200.07</td>
<td>85</td>
<td>2.58</td>
<td>2.587×10^{-11}</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>104,981,061,354</td>
<td>356,151.</td>
<td>128,300.53</td>
<td>104,981,061.351</td>
<td>2.48</td>
<td>2.371×10^{-11}</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>109,972,299,617</td>
<td>306,014.</td>
<td>134,399.21</td>
<td>109,972,299.614</td>
<td>2.78</td>
<td>2.531×10^{-11}</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Mean 2.59 \times 10^{-11}

4.2 Measurement of a THz test source

The bandwidth of the THz spectrum analyzer should be limited by that of the PCA including its antenna shape and phonon absorption. The spectral characteristics of the PCA as a THz detector have been previously investigated using THz time-domain spectroscopy, for which a bandwidth of 170 THz was achieved [12]. It is interesting to experimentally investigate the applicability of our spectrum analyzer to frequencies above 1 THz. Unfortunately, few test sources of CW waves are available in this frequency region. One source is the EM-THz comb radiating from a PCA emitter triggered by a fs-ML laser. Because the EM-THz comb is a group of many CW-THz waves having a frequency spacing equal to the laser mode-locked frequency, it can be used as a test source with multiple frequencies ranging from sub-THz to a few THz.

The generation and detection of EM-THz combs is given in detail elsewhere [9]. Therefore, only a brief description of the method is given here. We prepared another stabilized fs-ML Ti:sapphire laser (FEMTOLASERS Femtosource Scientific Pro) for a pump laser and a micro-structured, large-area GaAs PCA (GigaOptics Tera-SED 3) for generation of the EM-THz comb. In order to separate many beat signals generated between EM-THz comb mode and PC-THz comb mode, the mode-locked frequency of the pump laser \(f' = f' + \Delta f = 81,823,762\) Hz is slightly detuned from that of the probe laser \(f = 81,823,757\) Hz by an offset of \(\Delta f = 5\) Hz. The EM-THz comb having a frequency spacing of \(\Delta f = f'\) propagated in free space through pairs of spherical silicon lenses and off-axis parabolic mirrors, and was then focused onto a dipole-shaped LT-GaAs PCA for the THz spectrum analyzer, to improve the coupling efficiency of the EM-THz comb to the PCA. The resulting spectrum is shown in Fig. 6, in which the upper horizontal axis gives the frequency scale in the RF spectrum analyzer.

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[sweep time = 1000 s and RBW = 1 kHz]. The entire spectrum of the EM-THz comb is observed using a multi-frequency heterodyning photoconductive process occurring between the EM-THz comb (with a frequency of $f' = f + \Delta f$) and the PC-THz comb (with a frequency $f$) [9]. In this process, the observed RF spectrum is a replica of the EM-THz comb downscaled by a frequency factor of $f' / \Delta f = 16,364,752.4$. The actual frequency values in the EM-THz comb spectrum are given in the lower horizontal axis using the downsampling factor. The spectral bandwidth of the EM-THz comb extends over 1 THz. Since the THz electric field of our micro-structured, large-area PCA was comparable to that of usual bowtie-shaped LT-GaAs PCA (= 2 µW) [13], the average power per mode of the EM-THz comb (typically, less than 1 nW) is much lower than the average power of usual CW-THz sources. The EM-THz comb spectrum was clearly observed over 1 THz in spite of such low power, therefore, the THz spectrum analyzer has high sensitivity even in the THz region. The reason why the average power per mode of the detected EM-THz comb is below the detection power limit as mentioned in Sec. 4.1 (= 25 nW) is mainly due to difference of the sweep time of the RF spectrum analyzer.

![Spectrum of the EM-THz comb](image)

Fig. 6. Spectrum of the EM-THz comb. The upper and lower horizontal coordinates give the frequency scale in the RF and THz spectrum analyzers.

5. Conclusion

We have developed a THz spectrum analyzer that can be used to measure the absolute frequency and spectral shape of CW-THz sources. The proposed method is based on photoconductive heterodyning by combining a PCA as the receiver with a PC-THz comb as a local oscillator having multiple frequencies. Based on a stable THz frequency comb generated in a PCA, the absolute frequency of a sub-THz test source was determined at the precision of $2.8 \times 10^{-11}$. A 2.5-mW, 100-GHz CW test source was demonstrated, and an RF beat signal with a linewidth of 1.35 Hz was observed at a SNR of 50 dB, corresponding to a detection power limit of 25 nW. Furthermore, we confirmed that the spectral bandwidth of the THz spectrum analyzer extends over 1 THz using an EM-THz comb as a test source. Consequently, this method has the potential to become a powerful tool for practical CW-THz sources in real-world applications.

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