Direct Wavefront Measurement of Terahertz Pulses Using Two-Dimensional Electro-Optic Imaging

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Abstract—We report on the development of a wavefront sensor for terahertz pulses using a direct two-dimensional electro-optic imaging system composed of a ZnTe crystal and a CMOS camera. By measuring the phase variation of the terahertz electric field in the crystal plane, we are able to reconstruct the terahertz (THz) wavefront in order to determine the optical aberrations of the optical system. Associated with deformable mirrors or specifically designed optics, the sensor will open the route to terahertz adaptive optics.

Index Terms—Optical aberrations, sensor, terahertz (THz), wavefront.

I. INTRODUCTION

F rom the emergence of photonics technology, it has been forever essential to characterize and control the wavefront of optical beams, characterized by the surface of equiphase of the electromagnetic field, in order to determine the optical aberrations and optimize the performances of the optical systems. This can be accomplished for instance with the well-known Shack–Hartmann sensor which can locally measure the wavefront slopes of an incoming optical beam [1]. At present, in the visible and near-infrared spectral region, commercially available wavefront sensors can measure the optical aberrations and, associated with adaptive optics systems, they are able to correct these defaults in a wide variety of applications including astronomy, ophthalmology, and microscopy.

In the terahertz (THz) spectral domain, it is still challenging to measure the spatial profile and wavefront of a THz beam due to the lack of effective THz two-dimensional (2-D) detectors. However, the wavefront characterization of THz beams is essential to optimize the spatial resolution or illumination uniformity of THz imaging systems, developed for a large variety of applications such as security and defense, nondestructive testing of industrial materials, food and pharmaceutical industry, communications, or cultural heritage [2]. Wavefront analysis of THz pulses is also mandatory for the control of far-field intensity distribution of time-domain spectrometers or to increase the peak power of intense terahertz sources and optimize the beam fociation of scanning THz time-domain imaging systems.

Previous studies were oriented toward the measurement of THz wavefronts for both continuous wave and pulsed sources. In 2008, Bitzer et al. determined the beam profiles of THz pulses after passing through a hyper-hemispherical silicon lens [3]. They observed an asymmetric spatiotemporal electric field dynamic at the focus of the lens attributed to a distortion of the incident THz wavefront. However, they did not measure the THz wavefront, but only the THz beam profile. Moreover, their indirect and time-consuming method used a xy-scanning system in order to determine point-by-point the beam profile. In 2012, Cui et al. proposed the first THz wavefront measurement associated with a Hartmann sensor and indirect xy-scanning method, detecting the radiation with a pyroelectric sensor [4], [5]. The same year, Richter et al. presented a similar arrangement with a 2-D detection consisting in a microbolometer camera [6]. However, their method cannot be applied with short THz pulses produced by femtosecond laser pulses owing to the low sensitivity of the thermal incoherent detector. Another aperture scanning device was proposed in 2013 to characterize the THz beam propagation in a time-domain spectrometer [7]. Associated with a Hartmann mask, it has been used to produce 2-D image of the THz wavefront. More recently, we proposed the wavefront characterization of THz pulses using a Hartmann sensor associated with a 2-D electro-optic (EO) imaging system [8]. We quantitatively determined the deformations of planar and converging spherical wavefronts using the modal Zernike reconstruction least-squares method. Nevertheless, our indirect method uses a mask which limits the beam transmission and requires an initial calibration or reference measurement.

In this paper, we propose a new direct method for the wavefront measurement of THz pulses. The principle of our measurement is as follows. It is well known that spatiotemporal profiles of THz transient electric fields can be directly determined through EO sampling in a nonlinear crystal [9]. By varying the time delay between a short optical laser probe pulse and the THz pulse, EO sampling can record the spatial dependence of the full (amplitude and phase) THz electric field, $E_{THz}$, in the crystal plane. Then, from the phase variation of
$E_{\text{THz}}$ in this plane, we will demonstrate that a simple procedure makes it possible to determine the equiphase surface that is the wavefront.

II. EXPERIMENTAL DETAILS

For the demonstration of the method, broadband THz pulses were generated by optical rectification of amplified femtosecond laser pulses (800 nm, 1 mJ, 150 fs, 1 kHz repetition rate) in a 1 mm-thick ZnTe crystal with a clear aperture of 25 mm. The collimated THz beam is sent into another 1 mm-thick ZnTe crystal for collinear EO detection with a time-delayed femtosecond laser probe pulse, reflected by the front left face of the crystal (see Fig. 1). There, owing to the Pockels effect, the spatial distribution of the broadband THz electric field at the crystal position and finally determines the THz wavefront and extracts the optical aberrations.

III. RESULTS

A. Analysis of a Convergent THz beam

First, to implement our method, we introduced a plano-convex THz lens ($f = 100$ mm focal length) at $e = 65$ mm before the ZnTe crystal. This configuration provides a converging spherical THz beam at the crystal position with a theoretical $R = (f' - e) + \left(\frac{\pi w_0^2}{2}\right)^2/(\Lambda^2(f' - e)) = 38$ mm radius of curvature at 1 THz assuming an incident Gaussian beam and a $w_0 = 1$ mm beam-waist at the wavelength $\Lambda = 300$ µm.

Fig. 2(a) displays the distribution of the converging spherical THz electric field, $E_{\text{THz}}(x, y, \Omega = 1$ THz) in the ZnTe crystal plane. (c) Evolution of the relative phase along the horizontal black line indicated in (b).

From the data presented in Fig. 2, it is straightforward to plot the amplitude and the phase of the complex electric field $E_{\text{THz}}(\Omega)$, where $\Omega$ is the THz frequency [see Fig. 2(b)]. The THz spectrum extends from 0 to 3 THz with an expected linear dependence of the phase.

From the data presented in Fig. 2, it is straightforward to plot the amplitude and the phase of $E_{\text{THz}}(x, y, \Omega)$ for every pixels of the camera, as shown for $\Omega = 1$ THz in Fig. 3(a) and (b), respectively. For the sake of simplicity, the phase map at 1 THz plotted in Fig. 3(b) does not represent the absolute phase.
of the beam, but the relative one with a zero value arbitrary fixed at the center. Moreover, in the following of the paper, we will restrict our analysis to 1 THz but this latter can be easily generalized to every other frequency within 0–3 THz spectral range, emphasizing the ability of our sensor to characterize broadband THz pulses. Next, the final reconstruction of the THz wavefront at the crystal position will simply use the values of the relative phase of $E_{THz}(x, y, \Omega)$, represented by the phase map image in Fig. 3(b), as follows. First, we select a pixel $(x_0, y_0)$, e.g., the center of the phase map image, and we note the phase value $\phi_0(x_0, y_0)$ at this position. Then, except if the wavefront is perfectly planar without any optical aberrations, all the other pixels in the phase map image must have a different phase value. However, for every pixel position $(x, y)$, it is simple to calculate the distance $\Delta z(x, y)$ along the propagation axis for which the phase shift $\Delta \phi = 2\pi \Delta z / \Lambda$ will provide a new phase value being exactly $\phi_0$. The distribution of the resulting $\Delta z(x, y)$ values will design a new specific 3-D map, which will exactly correspond to the equiphase surface, demonstrating that our method is able to reconstruct the 3-D THz wavefront surface for every pixels of the camera.

To evaluate the accuracy of the method, we have to determine the minimal phase shift we can measure. Fig. 3(c) shows the evolution of the relative phase along the horizontal black line indicated in Fig. 3(b). The amplitude of the point-to-point fluctuations of the phase provides the minimum phase shift $\Delta \phi$ we can measure. For $\Omega = 1$ THz, we found $\Delta \phi = 100$ mrad, which corresponds to a minimum wavefront deformation of $(\Delta \phi \Lambda)/(2\pi) = 5 \mu \text{m}$, i.e., $\Lambda/60$. This wavefront measurement accuracy is comparable to standard commercial wavefront sensors designed for visible and infrared radiation.

First, the method is tested for the convergent THz beam, previously described in Figs. 2 and 3. Fig. 4(a) shows the reconstructed wavefront in the crystal plane with a quasi-spherical deformation from the center to the edge of the THz beam. The colored scale indicates the local wavefront deformation in $\mu \text{m}$. However, the precise observation of this surface indicates that the wavefront is not only governed by a single defocus but also by other optical aberrations since the surface does not exhibit a perfect circular symmetry. To quantify these aberrations, the standard method consists in developing the wavefront surface $W(x, y)$ onto a linear combination of the Zernike polynomials [11]

$$W(x, y) = \sum_i a_i Z_i(x, y)$$  \hspace{1cm} (1)

where $a_i$ represents the amplitudes of the polynomial $Z_i$, each polynomial corresponding to one specific optical aberration. Fig. 4(b) represents the amplitude of the 14 first Zernike polynomials. The main observation is obviously the strong defocus $a_4 = 350 \mu \text{m}$ (1.17$\Lambda$ at 1 THz). To compare this value to the theoretical $R = 38$ mm radius of curvature of the THz beam at the sample position, we can assume a perfect spherical wavefront (i.e., $a_i = 0, \forall i \neq 4$) and writes

$$W(r) = \frac{r^2}{2R} = 2\sqrt{3} a_4 \rho^2$$  \hspace{1cm} (2)

where $\rho = (x^2 + y^2)^{1/2}$ and $\rho = r/\alpha$. The parameter $\alpha$ is a normalization factor since the Zernike polynomials are defined for a maximum circular aperture with $\rho = 1$. In our measurement, we have $\alpha = 8.5$ corresponding to a 17 mm beam diameter. From (2), we calculate $R = 30$ mm which is about the theoretical value. The $20\%$ deviation can be attributed to the imperfect spherical wavefront. Fig. 4(b) can actually quantify these additional aberrations with a $x$-tilt $a_2 = 38 \mu \text{m}$ and a spherical aberration $a_{12} = -15 \mu \text{m}$. All the other aberrations are of the order of the measurement accuracy ($5 \mu \text{m}$), indicated by the dashed vertical black lines in the figure, and cannot be strictly considered as relevant in this experiment.

### B. Analysis of an x-Tilted THz Beam

After this first demonstration of validity of the method with a spherical wavefront, we removed the focusing lens before the detection crystal and intentionally horizontally tilted the THz beam with respect to the laser probe beam so that these two beams interact noncollinearly in the crystal. The angle between the two beams has been precisely measured to be $\theta = 3.9$ mrad. We introduced this precise tilt because it constitutes another optical aberration that we can finely control, providing another test to verify the validity of our method. According to the relation

$$W(r) = r \tan \theta = 2a_2 \rho$$  \hspace{1cm} (3)

the $x$-tilt parameter we introduced in the crystal plane is then $a_2 = 90 \mu \text{m}$. Fig. 5 shows the reconstructed wavefront and the amplitudes of the corresponding Zernike coefficients. As expected, the major optical aberration is the horizontal $x$-tilt with a calculated value $a_2 = 86 \mu \text{m}$ very close to the actual tilt we intentionally introduced. This additional test validates once again our wavefront reconstruction method using 2-D EO imaging.

### C. Analysis of a Planar THz Beam

Finally, a third experiment has been performed in order to determine the optical aberrations of the collimated THz beam.
If all optical aberrations are smaller than 20 µm, the THz wavefront is not exactly planar even if the THz wavefront is generated by optical rectification in the first ZnTe crystal. If the THz wavefront deformation arises from the laser beam or from the THz generation process in the crystal, we want to compare the measurement presented in this paper to that of [8], we need to propagate the THz electric field 10 mm before the crystal in order to reconstruct the THz wavefront at the same position. To this end, we have implemented a beam propagation method [12] to compute the field distribution at the position \( z_2 \), from the knowledge of the amplitude and phase of the field distribution at the position \( z_1 = z_2 + 10 \text{ mm} \). The relationship between both scalar fields is computed using the Helmholtz equation in free space to obtain

\[
E(x, y, z_2) = \mathcal{F}^{-1} \left[ E(k_x, k_y, z_1) \exp \left( \frac{k_x^2 + k_y^2}{2i\kappa_0} (z_1 - z_2) \right) \right]
\]

where \( E(k_x, k_y, z_1) \) is the Fourier transform of \( E(x, y, z_1) \), with \( k \) the wave vectors and the symbol \( \mathcal{F}^{-1} \) denotes the inverse Fourier transform. Note that the knowledge of both the amplitude and phase of the electric field at the position \( z_1 \) is necessary to back-propagate the field and compute \( E(x, y, z_2) \). Based on the precedent equation, a freely available MATLAB program has been computed in order to propagate the THz waves [13]. This procedure makes it possible to compare both methods. However, we want to stress again that, the direct method we propose in the present paper only relies on the phase distribution associated with the back propagation of 10 mm. Note that the accuracy of both the amplitude and phase of the electric field at the position \( z_1 \) is necessary to back-propagate the field and compute \( E(x, y, z_2) \). Based on the precedent equation, a freely available MATLAB program has been computed in order to propagate the THz waves [13]. This procedure makes it possible to compare both methods. However, we want to stress again that, the direct method we propose in the present paper only relies on the phase distribution of the THz electric field.

Let us recall that the Hartmann mask used in [8] consists of a 1-mm thick metallic plate with 9 by 9 circular holes (1 mm diameter with 2 mm periodicity in the horizontal and vertical directions). The accuracy of the Hartmann method depends on the design of the mask (distance between two holes and size of the holes). In our case, we estimated that the accuracy was equal to \( (\Delta/35) \) which is less precise than the direct measurement method proposed in this paper. In the case of the spherical convergent THz beam, we can see in Fig. 7 that the Zernike coefficients we obtain for the two methods are in good agreement. First, the amplitude of the defocus term is similar with \( a_4 = 270 \) µm with the Hartmann mask and \( a_4 = 310 \) µm with the direct method associated with the back propagation of 10 mm. Note that the decrease of \( a_4 \), i.e., the increase of the radius of curvature, from the position \( z_1 \) to the position \( z_2 \), is consistent with the convergent spherical THz wave. Both methods also indicate similar amplitudes for x-tilt and spherical aberration. Only the direct method is able to measure the 45 astigmatism with \( a_3 = -9 \) µm, since the value is lower than the accuracy of the Hartmann method, indicated by the dashed vertical black lines in the figure. It is
also important to notice that the Hartmann method measures the local slopes of the wavefront, which is finally calculated after integration of the data and application of a least-squares method [11]. Here, on the contrary, we directly measure the equiphasic surface of the THz beam, which constitutes the wavefront.

IV. CONCLUSION

In conclusion, by means of 2-D EO imaging of THz waveforms and subsequent calculation of the phase variation of the THz electric field, we were able to determine the wavefront with a precision of \( \lambda /60 \), without any prior reference measurement. We believe that our THz wavefront analyzer could provide an effective tool to control the generation of pulsed THz sources. Another interesting application concerns the association of the sensor with deformable mirrors in order to open the route to THz adaptive optics.

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REFERENCES


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